**ABD Journal, Volume 4 Number 1, 2012**

# A SIMULATION-OPTIMIZATION APPROACH TO THE INVENTORY MANAGEMENT OF TWO PERISHABLE AND SUBSTITUTABLE ITEMS

Bret R. Myers
Villanova University
Villanova, Pennsylvania, *USA*
bret.myers@villanova.edu

Avijit Banerjee
Drexel University
Philadelphia, Pennsylvania, *USA*
banerjea@drexel.edu

## Abstract

In this paper, a simulation-optimization approach is used to analyze an inventory policy for two perishable and substitutable items. A periodic review $(R,S_i)$ system of inventory control is considered, where demand for a preferred product can be satisfied by a substitute product with a known probability, in the event of a stockout of the former item. While taking demand substitution and finite product shelf life into account, the retailer is faced with the decision of determining the order-up-to level, $S_i$, for each product $i$ which maximizes expected total profit, given a common review period, R, determined exogenously.

## Introduction

This paper analyzes an inventory management policy for two substitutable and perishable items under demand uncertainty, using a simulation-optimization approach. Academics and practitioners alike are continually seeking ways to improve the management of perishable inventories. When considering the management of multiple items, product substitution is a possibility under many circumstances.

The scenario to be examined concerns the determination of an inventory policy for a retailer managing two perishable and substitutable items under a periodic review $(R,S_i)$ system of control. This type of system is typical in the retail grocery environment involving perishable items (Silver, 1998). Every $R$ units of time, a joint replenishment order occurs that brings the inventory position of each item $i$ up to a target level of $S_i$ units. In this study, $R$ is assumed to be exogenous and is predetermined based on existing conditions of the supply chain, while each $S_i$ level must be determined by the retailer. Our particular interest here is how the suggested $(R,S_i)$ policy is affected by changes in key factors such as the nature of product substitution, shelf-life, and the system cost structure.

Managing perishable inventory is a significant issue which affects many industries. Much attention has been focused on the grocery industry, where perishables account for over 50% of the $400 billion in annual retail sales in the US (Ferguson and Ketzenberg, 2005). There is

evidence that the selection of perishables available is the core reason why many consumers choose one supermarket over another (Heller, 2002). One study found that weekly sales are approximately 50% higher for perishable compared to non-perishable items (van Donselaar et al., 2005).

A major contribution of this paper is the incorporation of product substitution into inventory policy. Research shows that consumer-driven substitution due to product stockouts is a common dynamic in the grocery industry. The Grocery Manufacturers of America estimated that approximately 60% of consumers who experience a stockout, purchase a substitute product at the same store (Kraiselburd et al, 2004). Thus, it is important for managers to have an understanding of how product substitution can impact inventory policy. Van Donselaar et al. (2006) support this idea with the suggestion that a way to reduce waste is for the store manager to account for substitution in setting inventory control policies. Furthermore, it is believed that managers' inability to account for substitution is one of the contributing factors to the problems evident today in managing perishables across multiple industries.

The methodology in this paper entails the use of simulation-optimization in the determination of the inventory stocking levels that maximize expected profit. Simulation-optimization uses advanced search techniques such as genetic algorithms, simulated annealing, neural networks, scatter search, tabu search, etc. Consequently, candidate feasible solutions can be evaluated in an effort to obtain a best feasible solution after a specified number of runs (Law and Kelton, 2000). Through a numerical study, simulated-optimal solutions are obtained under varying operating conditions involving product substitution characteristics, shelf-life, cost structure and lead times.

This paper is organized as follows. Section 2 is a review of the existing literature that is relevant to managing perishable inventory with substitution for out-of-stock products. Section 3 details the model development for the maximization of expected profit under the $(R,S_i)$ inventory control system. Section 4 summarizes the results and analysis of a numerical study and finally, Section 5 summarizes the major findings of this work and suggests some areas of future research.

# Literature Review

Published research in perishable inventory dates back to the 1950s. Three comprehensive surveys that reflect the overall progression of perishable inventory research over the years are available in Nahmias (1982), Raafat (1991), and Goyal & Giri (2001). The coverage of perishable inventory literature includes problems dealing with the following features or characteristics: stock-dependent demand, price-dependent demand, permissible delays in payment, price changes, and time value of money. Overall, these three survey studies indicate that perishable inventory management has garnered much attention from researchers to date and that the classification system for this body of work has evolved and expanded over time.

The focus of this paper is on the management of perishable and substitutable items' inventories. Van Donselaar et al. (2006) indicate that little research has been done to date that combines product perishability and substitution. According to these authors Parlar (1985) is the only study that combines substitution with perishable items. Van Woensel et al. (2007) add that practitioners often fail to incorporate product substitution into inventory policy in managing

perishable grocery items. In the Parlar (1985) study, the nature of substitution considered is decision-maker-driven, where a manager has the option of swapping older items for fresh items. Examples mentioned include food stores, bakeries, and the apparel business. The problem involves a manager deciding how many units of fresh goods to supply and how many leftovers from the previous period can be substituted for the fresh items. This dynamic of substitution is consistent with the notion that while product utility can decrease with its age, consumers may be satisfied to purchase an older product instead of no product at all. An infinite horizon model is formulated and the optimal ordering policy is solved for a product that expires in two periods. Specifically, the problem is treated as a Markov decision problem by Parlar (1985).

A main reason for the dearth of research in combining substitution and perishability is that reordering becomes very complex, since the reorder quantity depends on detailed information about the number of items in stock for every age category. This makes the analysis of these systems quite cumbersome and the notion of substitution adds another layer of complexity. To overcome such difficulties in the formulation analytical models, a simulation-optimization approach is utilized in this paper.

Although little research combines product substitution and perishability, there is evidence of research where the simulation approach has been used for dealing with the case of either product perishability or substitution. Ferguson and Ketzenberg (2006), Broekmeulen and van Donselaar (2007), and Haijema et al. (2007) provide examples of using simulation to evaluate heuristics policies for inventory control of perishables. On the other hand, McGillivray and Silver (1978), Khouja et al (1996), Rajaram and Tang (2001), Shah and Avittathur (2006) and Kok and Fisher (2007) use simulation to evaluate heuristics for inventory problems involving product substitution. Haijema et al (2007) and Kok and Fisher (2007) are the only studies among these that utilize the technique of simulation-optimization.

Much research has been published on various classes of the single period (newsvendor) model that have also incorporated product substitution. Under a specific set of parameters, the newsvendor model is a special case of the problem detailed in this paper. Pasternack and Drezner (1991) consider a two product newsvendor problem with product substitution, where the probably of substitution in the event of a stockout is 1. Ernst and Kouvellies (1999) discuss the use of packaged goods as a substitute under the newsvendor settings for the two product variant case. Nangarajan and Rajagopalan (2008) derive the conditions where an optimal policy can be obtained for the two product case with product substitution in a single period model. These authors also provide a heuristic approach to a multi-period version of the same problem. What distinguishes the problem scenario in this paper from the Nagarajan and Rajagopalan (2008) study is that we classify substitution into age-based and product-based categories. Also, we handle product perishability differently from adopting a multi-period newsvendor type of model.

Another area of research that combines the newsvendor problem and product substitution is in the assortment planning problem. The assortment planning problem deals with determining inventory stocking levels of product variants under product substitution and shelf-space constraints. Smith and Agrawal (2000), Mahajan and Van Ryzin (2001), Kok and Fisher (2006), and Gaur and Honhon (2006) all contribute to the development of assortment planning models. However, these models do not specifically address product perishability beyond a single period and the aspect of age-based substitution introduced here.

# *Model Development*

## Problem Scenario and Assumptions

The problem in this paper involves a single retailer selling two product variants within an overall product class. Each product variant $i$ ($i$ = 1, 2) has a fixed shelf-life, $m_i$, which may or may not be equal for the two product variants offered. Once an item has reached its shelf-life, it is discarded, since it is no longer acceptable for sale. Substitution can take place among the two different variants within a product class (product-based), or between two different age classes for a particular product variant (age-based). For example, consider a customer shopping for lettuce. His/her first preference is Iceberg lettuce and second preference is Romaine lettuce. The customer will look for fresh Iceberg lettuce first. If there is no fresh Iceberg lettuce available, he/she could potentially choose among the following four alternatives:

1. Substitute with older Iceberg lettuce if it is available.
2. Substitute with fresh Romaine lettuce if it is available.
3. Substitute with old Romaine lettuce if no Iceberg lettuce is available
4. Decide to not to purchase any lettuce at all.

All substitution is considered to be consumer-driven. Choosing to substitute a fresh product with an older version of the same product is said to be age-based substitution, while choosing to substitute for a different product is termed product-based substitution.

For simplicity, each product will be classified as one of two age categories, namely, fresh and old. A similar approach to the classification of inventory by age is taken in Haijema et al (2007). A product is deemed fresh if its age is less than or equal to half its shelf-life. On the other hand, a product is old if its current age is more than half of the shelf life. For example, if a product has a shelf-life of 6 days then the product is fresh on the 1$^{st}$ 2$^{nd}$ and 3$^{rd}$ days on the shelf and old on the 4$^{th}$, 5$^{th}$, and 6$^{th}$ days on the shelf. We assume that the shelf-life of a product must be an even integer in order to fit into this age-based categorization framework. At the same time, it is assumed that both the retailer and the customer have complete knowledge of the shelf-life as well as the shelf age of a particular item.

Inventory is monitored periodically according to an ($R$, $S_i$) system of control. Every $R$ units of time, the inventory position of each product $i$ within a subcategory is reviewed, and an order is placed to bring its inventory position up to a level of $S_i$. $R$ is assumed to be exogenous while $S_i$ is endogenous. In other words, we assume that $R$ is given and predetermined based on properties of the relationship between retailers and other entities in the supply chain. With regard to ordering, a deterministic positive lead time is assumed for order arrivals, which is common to all items. We assume that ordering costs are independent of the size of the order, and are treated as a fixed cost. Therefore, since $R$ is fixed, order costs are treated as sunk costs and need not to be included in the expected profit function in order to determine the optimal $S_i$ values of each product. Holding costs are included in the expected profit function. Each item has a marginal holding

cost, $h_i$, applied to its average inventory level over a specified length of time, a unit purchase cost, $v_i$, and is sold at retail price of $p_i$.

The daily demand for an overall product class is assumed to be probabilistic, but based on a known distribution with known parameters. Each product *i* accounts for $\kappa_i$ proportion of the overall demand. For each product *i*, $\kappa_i$ is random and independent of overall demand for the product class. A similar model of demand for substitutable products within a subcategory is used in Nagarajan and Rajagopalan (2008). Daily demand for each product must be strictly integer valued.

The retailer must decide on the order-up-to level, $S_i$, for each variant within a product subcategory in order to maximize expected profits under demand uncertainty. The expected gross profit is the difference between the expected sales revenue and the total inventory related costs. No backorders are permitted, but apart from the opportunity loss, no additional penalty will be imposed on demand that is not satisfied (lost sales). Similarly, there is no additional penalty for expired items, i.e. the disposal costs are negligible.

## Product Sales and Substitution

A sale occurs if either a customer's initial preferred fresh product is available or if he/she willingly substitutes for either an older version of the same product or a different product. Each customer assigns a priority value {1,2,..n} to each of the available *n* products, 1 being the highest priority and n being the lowest priority, i.e., the most preferred product has a priority value of 1. For simplicity, we assume that each customer demands one unit of his/her preferred product. A customer's preferred product *i* is the product with a priority value 1. For simplicity, we limit the number of product variants considered in this study to two.

A customer wishes to purchase one fresh unit of their preferred product *i*. If there are no fresh units available in inventory of their preferred product, then the customer can have up to 3 options. If there are no fresh units available of their preferred product, then the customer can check the availability of old units of their preferred product, and check the availability of the other (*n-1*) products. When faced with available old units of the preferred product and available fresh units of a substitute product, the customer has 3 options. With a probability of $\alpha$, the customer will elect to substitute for one unit of fresh product *i* with one unit of old product *i*. With a probability of $\beta$, the customer will elect to buy one unit fresh product *j* as a substitute. Finally, with a probability of $(1- \alpha - \beta)$, the customer will elect not to make a purchase, which is referred to as a lost sale. If there is no substitute product *j* available and only old preferred product is in stock, then with a probability of $\omega$, the customer will choose to buy the old preferred product and with a probability of $(1-\omega)$, the customer will not make a purchase.

Another scenario is that there are neither fresh nor old units of the preferred product available. In this case, the customer has up to two options. If there are fresh units of a substitute product available, then with a probability of $\gamma$, the customer will elect to purchase one unit of the available fresh substitute product *j* and with a probability of $(1- \gamma)$, the customer will elect to leave without a purchase. If there are no fresh units of a substitute product available, but only old units of a substitute product are available, then the customer also will have two options. With a

probability of δ, the customer will elect to purchase 1 unit of old substitute product j and with a probability of (1- δ), the customer will leave without a purchase.

## Profit Function

Let $\pi_i$, representing the current cumulative profit for product *i*, be written as:

$$\pi_i = Z_i p_i - Q_i v_i - \bar{y}_i h_i. \tag{1}$$

where $Z_i$ represents the cumulative sales of product I, $p_i$ represents the market price for product variant *i*, $Q_i$ represents the quantity of product variant *i* ordered, $v_i$ represent the unit cost of product variant *i*, $\bar{y}_i$ represents the average inventory or product variant *i*, and $h_i$ represents the holding cost of product variant i per period. Thus, the overall total current profit for all n product variants, $\Pi$, is given by

$$\Pi = \sum_{i=1}^{n} \pi_i. \tag{2}$$

## Optimization Problem

The retailer is faced with the problem of choosing $S_i$ for each product *i* in order to maximize expected total profit under demand uncertainty. The overall product class demand X, is a random variable with p.d.f. of *f(x)* and parameters, $\mu$ and $\sigma$. Each product variant *i*, accounts for a proportion, $\kappa_i$, of the total demand for the product class. The order-up-to level for each product i, $S_i$, must be a non-negative integer. The resulting mathematical programming problem can be expressed as:

$$\underset{S_i \ \forall\, i=1\ldots n}{\text{maximize}}\ E(\Pi),$$

Subject to:

$$S_i \text{ is integer}, \ \forall\, i = 1 \ldots n, \tag{3}$$

$$S_i \geq 0\ , \qquad \forall\, i \in 1 \ldots n\ . \tag{4}$$

## Simulation-Optimization Model

Part of the proposed methodology in this paper involves the construction of a simulation-optimization model to find the best order-up-to level of inventory which maximizes expected profit in an ($R$, $S_i$) periodic review system. The first step of a simulation-optimization model is the design of the simulation model.

Let *l* be the length of a simulation run and *K* be the number of replications of a simulation run. The mean profit per simulation run of length *l* is estimated based on *K* replications and is expressed as follows:

$$\bar{\Pi}_l = \frac{1}{K}\sum_{r=1}^{K} \Pi_{r\cdot}\ , \tag{5}$$

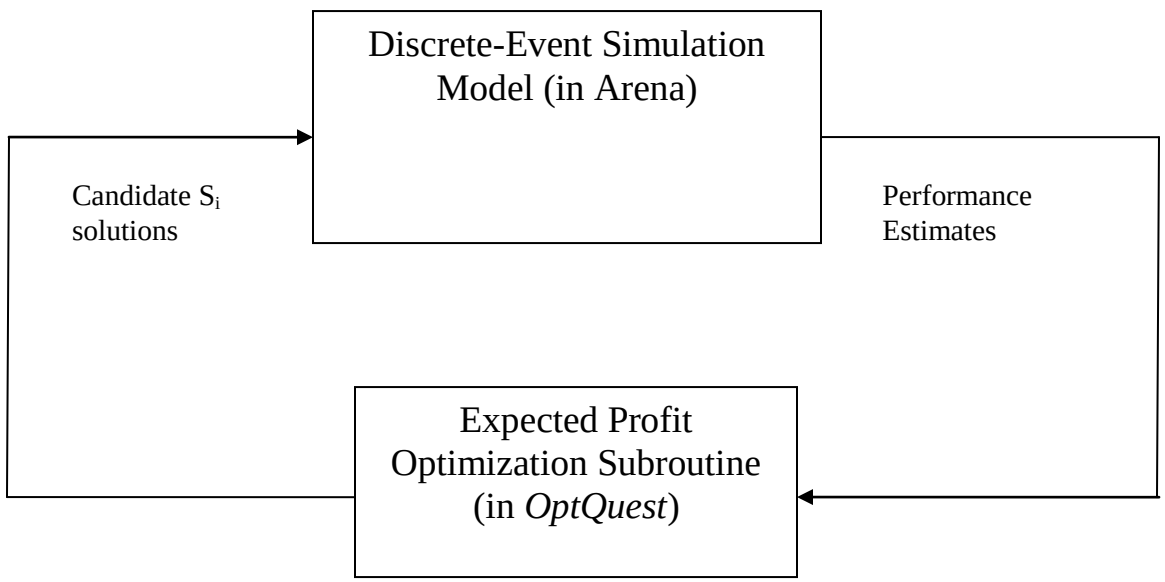
where $\Pi_r$ is the profit obtained from run $r$. The standard deviation of the profit per simulation run, $s_l$ is expressed as

$$s_l = \sqrt{\frac{1}{K-1}\sum_{k=1}^{K}(\Pi_k - \bar{\Pi}_l)^2} \ . \tag{6}$$

It is assumed that four major events occur over time, and the performance statistics will be collected after a certain warm-up period once the performance characteristics reach a steady state. The selection of replication length and warm-up period is based on Welch's graphical procedure (Law and Kelton 2000).

The simulation procedure is augmented through the addition of an optimization routine to a discrete-event simulation model. Candidate solutions are generated by efficient search techniques used in the optimization routine, and then the objective function values for the candidate solutions are evaluated after $K$ replications of the simulation model. *Arena* contains an optimization package called *OptQuest for Arena.* The overall simulation-optimization model is illustrated in Figure 1.

Figure 1: The Simulation-Optimization Model




The optimization routine is based on the problem of choosing the order-up-to level for each product that maximizes mean profit over $K$ of replications of the simulation of length $l$. The mathematical programming formulation for the simulation-optimization model is stated below:

$$\max_{S_i \ \forall \ i=1\ldots n} \bar{\Pi}_l, \tag{7}$$

Subject to:

$$S_i \text{ is integer}, \forall\, i = 1 \ldots n, \tag{8}$$

$$S_i \geq 0, \qquad \forall\, i \in 1 \ldots n\,. \tag{9}$$

The search for the optimal solution, executed via *OptQuest for Arena*, is based on a combination of three methods: scatter search, tabu search, and neural networks (Glover et al 1999). While attempting optimization through simulation, there may not be a guarantee of an optimal solution. In order for a solution to be considered optimal in the sense of stochastic optimization, it must produce an objective value that cannot be improved by any other candidate solutions in a statistically significant fashion.

# Results of Numerical Study

## Numerical Study Design

For the purposes of an experimental design for the simulation-optimization study, it is important to determine which parameters are to be fixed and which are to vary. The demand distribution, mean demand, coefficient of variation of demand, and variation of market share of each product will be held constant while the length of review period, lead time, shelf-lives of products, coefficient of variation of demand, proportion of overall demand for each product, price, holding cost, and substitution factors will all vary. Table 1 below provides a summary of the fixed and varied parameters, as well as the factors associated with our full factorial design.

Table 1: Fixed and varied parameters for simulation-optimization study

| **Fixed Input parameters** | **Level** |
|---|---|
| Demand distribution | Negative Binomial |
| $\mu_d$ | 22 |
| CV | 0.7 |
| **Factors** | **Levels** |
| R | 2,3 days |
| L | 0,1 days |
| $m_i$ | 4,6 days |
| $E(\kappa_i)$ | Equal, Unbalanced |
| $h_i$ | \$0.0083/unit/day, \$0.0833/unit/day |
| $p_i$ | \$4,\$5 |
| $v_i$ | \$1,\$2 |
| Product Based Substitution | High, Low |
| Age Based Substitution | High, Low |

The fixed parameters concern properties of product demand. The overall daily market demand will follows a negative binomial distribution with a mean of 22 and standard deviation of 0.7. The negative binomial distribution for demand is commonly used in the grocery industry (Ferguson & Ketzenberg 2006). The choice of parameters for the mean and the standard deviation of demand are arbitrary. The expected market share for each product will vary between 0.5 and 0.7; however, there is uncertainty in this parameter. A uniform distribution will be assumed for the market share of a product. In other words, the realized market share of a product can fall anywhere within 0.2 of the expectation in this two product case. For example, if the expected market share is 0.7 for a particular variant, then the realized market share can be anywhere from 0.5 to 0.9. This equates to a standard deviation of 0.033. This standard deviation of market share will remain constant throughout the study.

As for the varied parameters, the review period will be either 2 or 3 days and lead time will be zero or 1 day. However, only the following review period/lead time pairs are to be evaluated: (2,0), (2,1), and (3,0). Having (2,1) vs. (3,0) enables a comparative study on the effect of lead time on optimal stocking levels. Having (2,0) offers a smaller cycle length and allows for the examination of how optimal inventory policy is affected by increased shelf-life relative to the coverage period. Thus, it is decided that (3,1) is not necessary for inclusion in the simulation study as it will only yield marginal additional information.

The shelf-life of the products is varied between 4 and 6 days. It is possible for two product variants to have different shelf-lives. In all scenarios examined, the shelf-life of each product will exceed the cycle length. The maximum differential between product shelf-life and cycle length, therefore, is 4 days. Overall, there will be four age scenarios for each product variant as follows: (1) Scenario 1: both products have a shelf-life of 4 days, (2) Scenario 2: product 1 has a shelf life of 6 days and product 2 a shelf life of 4 days, (3) Scenario 3: product variant 1 has a shelf-life of 4 days and product variant 2 a shelf life of 6 days and (4) Scenario 4: both product variants have a shelf-life of 6 days.

In terms of substitution, there are two levels for both product-based and age-based substitution. A high level of product-based substitution with a high level of age-based substitution translates to $\alpha = 0.4$, $\beta = 0.5$, $\omega = 0.8$, $\gamma = 0.8$, and $\delta = 0.7$; a high level of product-based substitution with a low age-based substitution is depicted by $\alpha = 0.2$, $\beta = 0.7$, $\omega = 0.3$, $\gamma = 0.8$, and $\delta = 0.2$, a low level of product-based substitution with a high age-based substitution sets $\alpha = 0.7$, $\beta = 0.2$, $\omega = 0.8$, $\gamma = 0.3$, and $\delta = 0.2$, and a low level of product-based substitution with a low age based substitution is captured by $\alpha = 0.2$, $\beta = 0.2$, $\omega = 0.3$, $\gamma = 0.3$, and $\delta = 0.2$.

In terms of cost structure, there will be two scenarios for the marginal holding cost, $h_i$, applied to average inventory based on the length of the simulation run. One scenario is that the holding cost is relatively low, \$0.0083 per item per day and the other scenario is that the marginal holding cost is relatively high, \$0.0833 per item per day. These values are arbitrarily chosen and represent a stark contrast between what can be perceived as low versus high holding cost. The retail price will vary between \$4 and \$5 while the unit purchase cost will vary between \$1 and \$2. One requirement for the price and unit purchase cost is that the margin, $p_i$-$v_i$, must be the same for each product. Thus, the profit margin for each product will be \$2, \$3, or \$4. Overall, we examine four different cost scenarios. Each cost scenario is summarized in the table below:

**Table 2:** Cost Scenarios for Simulation-Optimization Study

| Cost scenario | $p_1$ | $v_1$ | $p_2$ | $v_2$ |
|---|---|---|---|---|
| **1** | 4 | 2 | 4 | 2 |
| **2** | 5 | 2 | 4 | 1 |
| **3** | 4 | 1 | 5 | 2 |
| **4** | 5 | 1 | 5 | 1 |

In cost scenarios 1 and 4, the product variants have identical retail price and unit costs, although, the values vary between scenarios. In cost scenarios 2 and 3, the retail price and unit costs differ, although the profit margin remains the same.

## Results

The Multiple Analysis of Variance (MANOVA) procedure was used to analyze the results of the simulation-optimization study for the order-up-to levels of two product variants. The two particular dependent variables of interest in the study are 1) the sum of the order-up-to levels, and 2) The differential between realized market share and expected market share for product variant 1. For example, assume that ordered pair for the optimal order-up-to levels for product variant 1 and 2 is (12,10). Also, we assume that the expected market share for product 1 is 0.60 and the expected market share for product 2 is 0.40. Dependent variable #1 would be the sum of the two products' order-up-to levels, i.e. 22 and dependent variable #2 would be -0.0545 since the realized market share for product 1 is 12/22 = 0.5454 and the expected market share is given as 0.60. From a managerial perspective, when stocking two products, it is important to be mindful of the overall sum of stock between the two products, and the allocation of the overall sum to each product.

Overall, the three factors identified for the study are level of substitution, cost structure, and shelf-life. As previously mentioned, there are four levels within each of these factors. It is of interest to identify whether or not significant effects exist on either of the dependent variables and also whether or not interaction effects exists when combining these factors. Table 2 below displays a summary of the statistical results.

Table 3: MANOVA Results for Main Effect and Interaction Effects involving

| **Main Effects** | | | | |
|---|---|---|---|---|
| | **Response Variables** | | | |
| | Sum | | Differential | |
| **Factor** | F | Significance | F | Significance |
| Substitution | 9.939 | 0.000 | 17.939 | 0.000 |
| Cost | 95.717 | 0.000 | 289.506 | 0.000 |
| Shelf-Life | 8.34 | 0.000 | 360.775 | 0.000 |
| **Interaction Effects** | | | | |
| | **Response Variables** | | | |
| | Sum | | Differential | |
| **Factors** | F | Significance | F | Significance |
| Substitution and Shelf-Life | 0.371 | 0.949 | 15.538 | 0.000 |
| Substitution and Cost | 90.309 | 0.972 | 18.868 | 0.000 |
| Shelf-life and Cost | 0.335 | 0.963 | 7.914 | 0.000 |
| Substitution, Cost, and Age | 0.059 | 1.000 | 1.051 | 0.396 |

These results reveal that for each of the factors of substitution, cost properties, and shelf-life has a significant impact on both the sum of the order-up-to levels for the two product variants and the differential between the actual allocation of that sum to each product variant and the expected allocation based on market share. In terms of interaction effects, it is revealed there are no combinations of factors that interact to affect the sum of the order-up-to levels; however, the following pairs of factors do interact to affect the difference in the allocation of the sum of the order-up-to levels and expected market share: substitution and shelf-life, substitution and cost, and shelf-life and cost.

As substitution turns out to have a significant impact on both the sum of the order-up to levels and difference in actual allocation vs. expected market share, it is also useful to discover which levels produce a statistically significant difference based on the Tukey HSD procedure. The results are summarized in Table 4 below:

Table 4: Tukey HSD procedure for Substitution Levels

| **Substitution Levels** | | | **Tukey HSD Subsets for Mean Response** | | | | |
|---|---|---|---|---|---|---|---|
| | | | **Response Variables** | | | | |
| | | | Sum | | | Differential | |
| Product-Based | Age-Based | N | 1 | 2 | 3 | 1 | 2 |
| Low | Low | 150 | 69.77 | | | 1.26 | |
| Low | High | 150 | | 76.24 | 76.24 | 1.42 | |
| High | Low | 150 | 72.45 | 72.45 | | | 3.69 |
| High | High | 150 | | | 77.04 | | 4.2 |

The above table indicates that age-based substitution plays a larger role in the effect on the sum of the order-up-to levels. There is a clear statistical difference when considering low product-based, low age-based vs. high product-based and high product-based, low age-based vs. high product-based, high-age based substitution. The evidence suggests that when age-based substitution is higher, the sum of the order-up-to levels increase. When considering the response of the difference in actual allocation of the sum of the order-up-to levels vs. expected market share, it is the level of product based substitution which plays the larger role. There are significant differences between high product-based, high age-based and low product-based, high age based, as well as between low product based, low age-based and high product based, low age-based substitution. There is evidence to suggest that when product-based substitution is high, there are incentives to stock at an offset from the designated market shares of each product. There is no significant difference between low product-based, low age-based and low product-based, high age-based, nor is there a significant difference between high-product based, high age-based and high product based, low age-based substitution.

# Conclusion

Overall, this paper provides a methodology for analyzing inventory policy for two substitutable and perishable product variants and yields the results of a numerical study involving a simulation-optimization procedure. From our findings, it is evident that substitution, shelf-life, and cost properties are all factors that tend to affect inventory policy. There is also evidence that the factor pairs of product substitution and cost structure, product substitution and shelf-life, and cost structure and shelf-life interact to affect the differential between actual allocation of the sum of the order-up-to levels and the expected market shares. Thus, managers can improve their inventory control policies by taking into consideration this new found evidence. Future research should focus on specific heuristics that account for these factors and their interactions which influence the response variables.

This paper also demonstrates how simulation can serve as both an evaluation tool and an optimizer. A simulation-optimization model incorporates all the complexities involving

production substitution and perishability that cannot be accounted for in a closed-form solution. As this paper deals with the two product variant case, future research will explore more than two product variants and to further develop heuristics for varying operating conditions.

One necessary step to enhance modeling of perishable and substitutable items is to collect data for a better understanding of the nature of product and estimation of the relevant parameters. This study assumes that there are two types of substitution: product-based and age-based. Each of these factors respectively played a role in shaping inventory policy. Thus, it is important that in practice these parameters be accurately estimated. Practitioners can collect the necessary data through consumer surveys and observational studies. Finally it is hoped that the findings of our study represents a step forward in developing a better understanding of complex real world scenarios and overcoming the practical difficulties associated with managing perishable and substitutable items.

# References

Broekmeulen, R., K, van Donselaar. A replenishment policy for a perishable inventory system based on estimated aging retrieval behavior. Working paper, Technische Universiteit Eindhoven, Eindhoven, The Netherlands 2007.

Ernst, R., P. Kouvelis. The effects of selling packaged goods on inventory decisions. *Management Science* 1999; 45(8): 1142-1155.

Ferguson N, Ketzenberg ME. Informations Sharing to Improve Retail Product Freshness of Perishables. *Production and Operations Management* 2006; 15(1): 57-73.

Gaur, V., D. Honhon. Assortment planning and inventory decisions under a locational choice model. *Management Science* 2006; 52(10): 1528-1543.

Goyal SK, Giri BC. Recent trends in modeling of deteriorating inventory. *European Journal of Operations Research* 2001; 134: 1-16.

Haijema R, Wal J van der, Dijk NM van. Blood platelet production: Optimization by dynamic programming and simulation. *Computers & Operations Research* 2007; 34(3): 760-779.

Heller, T. Sales grow so does competition. *Progressive Grocer* 2002 81(10) 103-106.

Khouja M., A. Mehrez, G. Rabinowitz. A two-item newsboy problem with substitutability. *International Journal of Production Economics* 1996; 44: 267-275.

Kök, A.G., M. Fisher. Demand estimation and assortment optimization under substitution: methodology and application. *Operations Research* 2007; 55(6): 1001-1021.

Kraiselburd, S., Narayanan, V.G., and Raman, A. Contracting in a supply chain with stochastic demand and substitute products. *Production and Operations Management* 2004; 13(1): 46-62

Law AM, Kelton WD. *Simulation modeling and analysis* (third ed.). Boston: McGraw-Hill, 2000.

Mahajan, S., G.J. van Ryzin. . Stocking retail assortments under dynamic consumer substitution. *Operations Research* 2001. 49(3): 334-351.

McGillivray, A.,E.A. Silver. 1978. Some concepts for inventory control under substitutable demand. *INFOR* 16(1) 47-63.

Nagarajan, M., S. Rajagopalan. Inventory models for substitutable products: optimal policies and heuristics. *Management Science* 2008; 54(8): 1453-1466.

Nahmias S. Perishable inventory theory: a review. *Operations Research* 1982; 30(4): 680-708.

Parlar, M. Optimal ordering policies for a perishable and substitutable product—A Markov decision model. *INFOR* 23: 182-195.

Pasternack, B., Z. Drezner. Optimal inventory policies for substitutable commodities with stochastic demand. *Naval Research Logistics* 1991 38(2): 221-240.

Raafat F. Survey of literature on continuously deteriorating inventory. *Journal of Operations Research Society* 1991;42: 27-37.

Rajaram, K.C., S. Tang. The impact of product substitution on retail merchandising. *European Journal of Operations Research* 2001; 135(3): 582-601.

Shah, J., and B. Avittathur. The retailer multi-item inventory problem with demand cannibalization and substitution. *International Journal of Production Economics* 2007; 106: 104-114.

Silver EA, Pyke DF, Peterson R. Inventory Management and *Production Planning and Scheduling* (third ed.). Chichester: Wiley, 1998.

Smith, S.A., N. Agrawal. Management of multi-item retail inventory Systems with demand substitution. *Operations Research* 2000; 48(1): 50-64.

Van Donselaar K., T. van Woensel, R. Broekmeulen, J. Fransoo. Inventory control of perishables in supermarkets. *International Journal of Production Economics* 2006; 104: 462-472.

Van Woensel, T., K van Donselaar, R. Broekmeulen, J. Fransoo. Consumer responses to shelf-out-of stock perishable products. *International Journal of Physical Distribution & Logistics Management* 2007; 37(9): 704-718.

## Biographies

**Bret Myers** is an assistant professor of management and operations at the Villanova School of Business. He received his Ph.D. in Decision Sciences from Drexel University in 2009. Prior to his graduate studies, Bret played professional soccer for 2 seasons with the Richmond Kickers. His research interests are in in the general area of operations and supply chain management and also concerning the use of analytics in business and sports. Bret's research in the area of sports analytics has twice been featured in the *Wall Street Journal* and also mentioned in the *New York Times*.

**Avijit Banerjee** is a professor of operations management in the Decision Sciences Department in the LeBow College of Business of Drexel University. He obtained his Ph. D. in Production/Operations Management from the Ohio State University. His current research interests include supply chain management, inventory models and operations scheduling. His numerous research articles appear in several scholarly journals, such as *Management Science*, *Decision Sciences*, *International Journal of Production Economics*, *Omega* and *International Journal of Operations and Production Management*. He is also an active member of INFORMS, DSI, POMS, IIE and ISIR.