# A Goal Programming Approach to a Media Mix Advertising Plan: A Case Study of a Private-Practice Health Care Provider 

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#### Abstract

This case study designs and builds an operational model for media mix determination for a private-practice physician's office. The recommended advertising plan is determined via goal programming in order to balance the key issues of size-of-audience reached versus annual and quarterly advertising budgets. Post-optimality / sensitivity analysis is conducted on selected parameters of the model in order to gauge the robustness of the recommended media mix.


## Introduction and Purpose

It has been more than 30 years since the use of media advertising by professional service providers (accountants, attorneys, physicians) was formally sanctioned. Historically, many professional associations had prohibited their licensed members from engaging in speech activities that suggested a commercial transaction i.e. advertising [10]. They argued that advertising would have an adverse impact on the image of their profession. Typically, such opposition to physician advertising has focused on such image and ethical issues, while current arguments favoring physician advertising focus on information needs, revenue and competitive issues.

The current status of professional-services advertising came about as a result of the application of existing anti-trust laws and an interpretation of First Amendment to commercial speech. In the mid-‘ 70 s, the anti-trust laws were used to strike down the anti-competitive "Code of Ethics", which had allowed professional associations to discipline members who advertised. A landmark Supreme Court ruling [6] established that the anti-trust laws did indeed apply to the 'learned professions' and such professions
were engaged in 'commerce'. By 1977, the Supreme Court had further ruled that commercial speech was a protected form of expression under the First Amendment; thus states and agencies empowered to act for states (such as professional licensing boards) were prohibited from banning advertising by practicing professionals [16, 2]. The Federal Trade Commission found the American Medical Association had indeed caused substantial harm to the public by restricting advertising by physicians because such restrictions "...served to deprive consumers of the free flow of information about the availability of health care services..." and had suppressed "...the offering of innovative forms of health care..." [1]. The FTC decision was not intended to compel physicians to advertise; rather, it was merely intended to modify the AMA's code of ethics to permit enough advertising to provide patients with "a decisional basis for selecting one doctor as opposed to another."

Today private practice health care providers, like most businesses, will use advertising in order to attract potential customers to its products or services. Media selection involves the allocation of an advertising budget among a variety of media options in such a way as to maximize the number of potential customers reached [12]. Both linear programming and goal programming, have been applied to the purpose of selecting media purchases. In this case study, a particular physician's office is seeking to design and implement an advertising campaign that will enable it to reach the greatest number of potential patients, consistent with its budget. In the past, the media planner simply assessed the available advertising options and chose a mix based upon the subjective analysis, often resulting in a costly advertising program that yielded very small returns. The purpose of this paper is to apply goal programming, as a tool for managing the relationship between exposures and expenses, in determining the optimal selection of media mix for the private-practice office.

## Literature Review

Over the last three decades, advertising has become a key component of the business plan of many professionals, who confront many of the same business issues faced by traditional firms, including increased competition, greater knowledge and demands by customers (patients), and a very dynamic economic environment. Interestingly, the yellow pages became the early leader among the advertising media used by professional service providers; Marks and Moon [9] found printed media to be highly preferred to other media. Many studies have sought to assess the evolving attitudes of either customers or providers towards professional-service advertising [4, 9, 10, 11]. They conclude that consumers generally have a positive attitude toward professional advertising, as they believe that it provides them with useful information for making better choices. In a study of self-employed physicians, Rizzo [13] found a comparatively high level of advertising among plastic surgeons relative to other specialists. Moser [11] examined whether age, occupation, income, education, and/or gender of patients account for any significant difference in attitudes toward physicians who advertise. His longitudinal study replicated a similar study done 20 years earlier, and concluded that, while the ethical debate continues, advertising certainly has a constructive place in the health care industry.

In recent years, physicians have become active in marketing their services to the public and conducting such advertising via a number of media including television, yellow pages, highway billboards, web sites, and even e-mail [3]. Recent studies show most physicians who advertise will witness a resulting increase in patient flow and net revenue, even in an increasingly competitive environment [8, 15]. Freedman [5] found that the return on dollars invested by physicians in advertising was four to six times the cost. Consequently, a marketing budget has become critical for most medical practices. Many health care professionals now use marketing consultants or have their own internal marketing/advertising committees [14].

In the optimization literature relating to advertising models, Rifai and Hanna [12] discussed a linear programming model that involves the setting of goals in terms of exposures, reach, frequency of exposures, and frequency distribution of the number of occasions of exposure. Given these goals, their model sought to select the best, or close to the best media portfolio, rather than one based only upon maximizing exposure.
Kluyver [7] pointed out that, when formulating maximum effective exposure or weighted exposure schedules, "optimum levels of insertion cannot be obtained for all media without considerable effort (and cost), and boundaries of acceptability are used instead (pp.27)." In other words, it is not always possible or practical to achieve an exact goal, so parameters are established within which the solution is acceptable. The Media Mix Project of this study follows Kluyver's model by using hard constraints to designate the acceptable parameters and using soft constraints to indicate the most desirable goals.

## Assumptions

The data regarding the various media options were obtained from the media sources themselves. It is assumed that the data is true and accurate. Most of the data given was measured monthly, so the model relies upon the ability to extrapolate the data over a three-month period and over a 12 -month period.

A significant assumption made in the model formulation is that the past number of exposures from the different media alternatives will remain consistent for the next year. The model also assumes that each exposure on a potential customer has a constant impact, yielding consistent response rates from the audience. Audience duplication and repetition are not taken into consideration in the model, although repetition is typically good.

An annual budget is given for the project, and it is assumed that the yearly budget can be divided evenly among each quarter as no seasonal pattern is presumed.

## Variable Explanation

The control variables in this case study are the various media alternatives, from which the physician's office must choose, in order to develop a local advertising campaign. The variables consist of the number of ads placed with specific local media during each of the four quarters of the next fiscal year. The candidate media include local radio stations,
local periodical print media (newspapers and magazines), Internet website listings, church bulletins, and the local philharmonic publication. Most variables will be restricted to be general counting integers, while some are simple binary variables. Tables 1 through 6, included in Appendix A, provide a comprehensive list of the variables. The variable names chosen attempt to be descriptive of the particular associated media; in all cases, the last character of the variable name identifies the relevant fiscal quarter, 1 through 4. The model is built with 18 possible media and four quarters, and consequently, a total of 72 variables.

The various media sources were surveyed to determine the number of exposures per month (extrapolated over a three-month period) and the cost per ad, displayed in Table 7 below. Because the number of exposures and the cost per ad remains constant for each quarter for every media source except the Philharmonic books, only the data for Quarter 1 is shown in the table. The same values were used for Quarters 2, 3, and 4.

TABLE 7: Media Alternatives

| Media | Total \# of <br> Exposures per <br> quarter | Cost (\$) <br> per Ad | Media | Total \# of <br> Exposures <br> per quarter | Cost (\$) <br> per Ad |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| WAJI1 | 170070 | $\$ 37.50$ | FWPAR1 | 52800 | $\$ 695.00$ |  |  |
| WLDE1 | 181914 | $\$ 39.87$ | YELLOF1 | 669 | $\$ 40.00$ |  |  |
| FWND1 | 290400 | $\$ 148.75$ | SUPER1 | 1167 | $\$ 123.00$ |  |  |
| FWNS1 | 842700 | $\$ 214.10$ | FWWEB1 | 24000 | $\$ 600.00$ |  |  |
| ABNE1 | 59400 | $\$ 254.00$ | STVIN1 | 3300 | $\$ 160.00$ |  |  |
| TIMDUP1 | 57000 | $\$ 254.00$ | LADY1 | 2400 | $\$ 287.00$ |  |  |
| FWNM1 | 97158 | $\$ 348.03$ | PHB1 | 6000 | $\$ 1,205.00$ |  |  |
| NILM1 | 156000 | $\$ 1,215.00$ | PHB2 | 4300 | $\$ 860.00$ |  |  |
| BUWM1 | 195900 | $\$ 1,000.00$ | PHB3 | 4200 | $\$ 860.00$ |  |  |
| MDNM1 | 12000 | $\$ 630.00$ | PHB4 | 5000 | $\$ 420.00$ |  |  |
| FWMON1 | 60000 | $\$ 1,185.00$ |  |  |  |  |  |

This case will set budgetary targets for the planning year and each component quarter; these financial expense targets can be underachieved without concern. Goal Programming will be used to provide for reluctant overachievement of the budgetary targets should such be warranted. This will necessitate the need for a pair of deviation variables in each budgetary constraint. The notation of " $d_{\text {targetID }}$ plus' and ' $d_{\text {targetID }}$ minus' will be used to designate the ten deviation variables for the five budgetary targets/goals.

## Constraint Development

In a linear programming model, formal constraints capture the environment that the decision maker confronts. Such constraints may limit or prevent certain things from happening and force other things to happen. This case study has the following constraints:

## Budgets

The pre-determined budget allocated for advertising expenditure is $\$ 30,000$ for the year. Given the absence of seasonality, the annual budget is divided equally among the quarters; thus, the quarterly advertising budget is $\$ 7500$. The following constraints describe the budget for Quarter 1, Quarter 2, Quarter 3, Quarter 4, and the annual budget, respectively. As acknowledged earlier, deviation variables were added to the budget constraints to allow for some flexibility and to not rule out an ideal media mix that oversteps the budget by only a small amount.
BDG1) 37.50WAJI1 + 39.87WLDE1 + 148.75FWND1 + 214.10FWNS1 +
254ABNE $1+254$ TIMDUP1 + 348.03FWNM1 + 1215NILM1 + 630MDNM1 +
1000BUWM1 + 1185FWMON1 + 695FWPAR1 + 40YELLOF1 + 123SUPER1
$+600 \mathrm{FWWEB} 1+160$ STVIN $1+287 \mathrm{LADY} 1+1205 \mathrm{PHB} 1+$ d1minus -d 1 plus $=$
7500
BGT2) 37.50WAJI2 + 39.87WLDE2 + 148.75FWND2 + 214.10FWNS2 +
$254 \mathrm{ABNE} 2+254$ TIMDUP $2+348.03$ FWNM $2+1215$ NILM $2+630$ MDNM $2+$
1000BUWM $2+1185$ FWMON $2+695$ FWPAR $2+40$ YELLOF $2+123$ SUPER 2
+600 FWWEB2 +160 STVIN $2+287$ LADY2 +860 PHB $2+$ d2minus - d2plus $=$
7500

BGT3) 37.50WAJI3 + 39.87WLDE3 + 148.75FWND3 + 214.10FWNS3 + $254 \mathrm{ABNE} 3+254$ TIMDUP3 +348.03 FWNM $3+1215$ NILM $3+630$ MDNM $3+$ 1000BUWM3 + 1185FWMON3 + 695FWPAR3 + 40YELLOF3 + 123SUPER3 +600 FWWEB3 +160 STVIN3 +287 LADY3 $+420 \mathrm{PHB} 3+$ d3minus - d3plus $=$ 7500
BGT4) 37.50WAJI4 + 39.87WLDE4 + 148.75FWND4 + 214.10FWNS4 + 254ABNE4 + 254TIMDUP4 + 348.03FWNM4 + 1215NILM4 + 630MDNM4 + 1000BUWM4 + 1185FWMON4 + 695FWPAR4 + 40YELLOF4 + 123SUPER4 + 600FWWEB4 + 160STVIN4 + 287LADY4 + 420PHB4 + d4minus - d4plus $=$ 7500
BGTANN) 37.50WAJI1 + 37.50WAJI2 + 37.50WAJI3 + 37.50WAJI4 + 39.87WLDE1 + 39.87WLDE2 + 39.87WLDE3 + 30.87WLDE4 + 148.75FWND 1 +148.75FWND2 + 148.75FWND3 + 148.75FWND4 + 214.10FWNS1 + 214.10FWNS2 + 214.10FWNS3 + 214.10FWNS4 + $254 \mathrm{ABNE} 1+254 \mathrm{ABNE} 2+254 \mathrm{ABNE} 3+254 \mathrm{ABNE} 4+254 \mathrm{TIMDUP} 1+$ 254 TIMDUP $2+254$ TIMDUP $3+254$ TIMDUP $4+348.03$ FWNM $1+$ 348.03FWNM $2+348.03$ FWNM3 + 348.03FWNM4 + 1215NILM $1+$ 1215NILM2 + 1215NILM3 + 1215NILM4 + 630MDNM1 + 630MDNM2 + 630MDNM3 + 630MDNM4 + 1000BUWM1 + 1000BUWM2 + 1000BUWM3 + 1000BUWM $4+1185$ FWMON $1+1185$ FWMON $2+1185$ FWMON $3+$ 1185 FWMON $4+695$ FWPAR $1+695$ FWPAR $2+695$ FWPAR3 + 695FWPAR 4 + 40YELLOF1 + 40YELLOF2 + 40YELLOF3 + 40YELLOF4 + 123SUPER1 + 123SUPER2 + 123SUPER3 + 123SUPER4 + 600FWWEB $1+600$ FWWEB $2+$ 600FWWEB3 + 600FWWEB4 + 160STVIN1 + 160STVIN2 + 160STVIN3 + 160STVIN4 + 287LADY1 + 287LADY2 + 287LADY3 + 287LADY4 + $1205 \mathrm{PHB} 1+860 \mathrm{PHB} 2+420 \mathrm{PHB} 3+420 \mathrm{PHB} 4+\mathrm{d} 5$ minus -d 5 plus $=30000$

## Media Usage Restriction

Decision models are typically designed to include 'policy' constraints, which reflect the decision maker's imposition on latitude provided for the ultimate recommendation. Forced diversification constraints typically fit that description, whether in financial portfolio models or advertising mix models. In this case, the physician's office has set the policy that no more than $20 \%$ of the annual budget should be used on any single media category. Since the 18 media options fall into six media categories, (newspaper, magazine, radio, Internet, church bulletin, and philharmonic books), six constraints will be used to enforce this desire for diversification. As representative of these constraints, the following enforces that cap for radio advertising collectively over the two targeted local stations (WAJI and WLDE):

$$
\begin{aligned}
& \text { RADBGT) 30WAJI1 + 30WAJI2 + 30WAJI3 + 30WAJI4 + 31.9WLDE1 + } \\
& \text { 31.9WLDE2 + 31.9WLDE3 + 31.9WLDE4 - 29.75FWND1 - 29.75FWND2 - } \\
& \text { 29.75FWND3 - 29.75FWND4 - 42.82FWNS1 - 42.82FWNS2 - 42.82FWNS3 - } \\
& \text { 42.82FWNS4 - 82FWNS4 - 50.8ABNE1 50.8ABNE2 50.8ABNE3 - } \\
& \text { 50.8ABNE4 - 50.8TIMDUP1 - 50.8TIMDUP2 - 50.8TIMDUP3 - } \\
& \text { 50.8TIMDUP4-50.8TIMDUP4-69.61FWNM1 - 69.61FWNM2 - } \\
& \text { 69.61FWNM3 - 69.61FWNM4 - 243NILM1 - 243NILM2 - 243NILM3 - } \\
& \text { 243NILM4 - 126MDNM1 - 126MDNM2 - 126MDNM3-126MDNM4 - } \\
& \text { 200BUWM1-200BUWM2-200BUWM3-200BUWM4-237FWMON1 - } \\
& \text { 237FWMON2-237FWMON3-237FWMON4-139FWPAR1-139FWPAR2 - } \\
& \text { 139FWPAR3-139FWPAR4-8YELLOF1-8YELLOF2 - 8YELLOF3 - } \\
& \text { 8YELLOF4 - 24.6SUPER1 - 24.6SUPER2 - 24.6SUPER3 - 24.6SUPER4 - } \\
& \text { 120FWWEB1 - 120FWWEB2 - 120FWWEB3 - 120FWWEB4 - 32STVIN1 - } \\
& \text { 32STVIN2-32STVIN3 - 32STVIN4 - 57.4LADY1 - 57.4LADY2 - } \\
& \text { 57.4LADY3-57.4LADY4-241PHB1 - 172PHB2-172PHB3-84PHB4 <= } 0
\end{aligned}
$$

## Newspaper Availability

The local newspapers Aboite \& About and Dupont Valley Times only have 3 publications per quarter (1 per month). Therefore, the following trivial constraints were used for Aboite \& About and for Dupont Valley Times for each quarter, respectively.

```
ABNEAV1) ABNE1 <= 3
TIMDUPA1) TIMDUP1 <= 3
ABNEAV2) ABNE2 <= 3
TIMDUPA2) TIMDUP2 \(<=3\)
ABNEAV3) ABNE3 <= 3
TIMDUPA3) TIMDUP3 <= 3
ABNEAV4) ABNE4 <= 3
TIMDUPA4) TIMDUP4 <= 3
```


## Magazine Requirement

Past results from advertising in local magazines have been quite positive. Consequently, another policy constraint is to always run at least one but no more than three magazine ads per quarter, so as not to over-saturate the market. Given the six distinct magazine candidates, the following constraints reflect the allowable window of magazine ads for the first quarter; the other quarters are handled in parallel fashion:

```
MAGMIN1) FWNM1 + NILM1 + BUWM1 + MDNM1 + FWMON1 +
FWPAR1 >= 1
    MAGMAX1) FWNM1 + NILM1 + BUWM1 + MDNM1 + FWMON1 +
FWPAR1 <= 3
```


## Church Bulletin Requirements

The physician is personally involved in two local church congregations (designated STVIN and LADY) and has requested that the possibility of advertising in the churches' weekly bulletins be considered in the model. For political reasons, he established the requirement that if either church bulletin is used then both church bulletins must be used. In other words, an ad must run in either both church bulletins or in neither. The following constraint enforces this policy for Quarter 1; the other quarters have parallel treatment:

CHB1) STVIN1 - LADY1 $=0$

## Media Diversity Requirements

Past experience in advertising has demonstrated that patients are attracted to the practice through a variety of media. Another policy to maintain a diverse advertising plan is to place at least one ad in each media category per quarter, with the exception of the church bulletins. The following constraints construct this requirement for the newspaper media category (with four paper options) for each of the four quarters; the other categories are treated in parallel fashion. (A constraint for a minimum of one magazine ad per quarter was previously established in the Magazine Requirement above.)

NEWREQ1) FWND1 + FWNS1 + ABNE1 + TIMDUP1 >= 1
NEWREQ2) FWND2 + FWNS2 + ABNE2 + TIMDUP2 >= 1
NEWREQ3) FWND3 + FWNS3 + ABNE3 + TIMDUP3 >= 1
NEWREQ4) FWND4 + FWNS4 + ABNE4 + TIMDUP4 >= 1

## Philharmonic Contract Requirements

Advertising contracts with the Philharmonic state that advertisers may choose between two options: to purchase a full season plan (placing an ad in all 4 books) or a half season plan (placing an ad in books $1 \& 2$ or books $3 \& 4$ ). It was decided to also limit advertising to just one ad per book. The following constraints limit the selection to one ad per book for books $1 \& 2$, ensures that books $1 \& 2$ are linked together. Two similar constraints perform this linking function for books $3 \& 4$.

PH1) PHB1 + PHB2 $<=2$
PHS1) PHB1 - PHB2 $=0$

## Binary Constraints

Selected variables in the model are binary; the variable is either on or off. When the variable is on, it also means that one ad will be used. For example, for YellowBook.com and for Superpages.com only one listing is necessary per Internet site (similar to the
yellow page alphabetical listings in a traditional telephone directory). More than one listing would not make sense. And while the physician requested that the church bulletins be taken into consideration for the advertising plan, he also stated that at most only one ad should be used for each church bulletin. Finally, the Philharmonic books are only printed quarterly, and it was previously established that only one ad may be used per book. In sum, five distinct media alternatives for each of four possible quarters (i.e. twenty of the original 72 decision variables) were restricted to binary values.

## Non-Negativity

The non-negativity constraints prevent the occurrence of an impractical solution, such as running a negative number of ads each quarter.
In sum, the model has a total of 47 constraints (not including non-negativity constraints), 39 of which are non-trivial constraints. The entire model can be found in Appendix B.

## Objective Function

Goal programming was used to solve the model so that two goals with different priorities could be achieved. The two goals that the model sought to achieve were:

1) minimize the overachievement deviations of the quarterly and annual budget constraints, and 2) maximize the number of exposures generated from the media selection.

The first goal was achieved by running the model with the following objective function:
Minimize $\sum$ overachievement deviations of the five budget targets (subject to the previously discussed constraints)
Minimizing the overspending for each quarter is important, but it is more critical that the annual budget was not exceeded. Therefore, because keeping to the annual budget was determined by the physician to be twice as important as keeping to each individual quarterly budget, a penalty weight coefficient of 2 was used for the deviation variable associated with the overachievement of the annual budget (i.e. d1plus + d2plus + d3plus +d 4 plus +2 d 5 plus)

The second goal was to maximize the annual number of exposures generated from the media selection. The total exposures from each media alternative for each quarter were added together in the following objective function:

> Maximize $\sum$ exposures from media alternatives
> (subject to the previously discussed constraints)

In order to enforce the first priority objective function (minimizing the overachievement of the quarterly and annual budgets) a constraint was added for the second objective function (i.e. G1) d1plus +d 2 plus +d 3 plus +d 4 plus +2 d 5 plus $=$ optimal solution value from initial run).

## Conceptual Model

The goal of the advertising campaign was to maximize the total number of annual exposures generated from an advertising plan while also minimizing overspending of the budget, subject to: annual and quarterly budgets and media usage requirements and limitations.

Using Goal Programming, the model appeared as:
Minimize: the overachievement of Quarter 1 budget, plus the overachievement of Quarter 2 budget, the overachievement of Quarter 3 budget, the overachievement of Quarter 4 budget, the overachievement of the annual budget (which is twice as important as each quarterly budget)

## Subject to:

$\sum \operatorname{cost}_{Q 1}$ does not exceed Quarter i budget; $\mathrm{i}=1-4 \quad$ (yet overachievement is reluctantly permitted)
$\sum \operatorname{cost}_{Q 1, Q 2, Q 3, Q 4}$ does not exceed Annual budget; (yet overachievement is reluctantly permitted)
$\sum \mathrm{ad}_{i}$ does not exceed $20 \%$ of budget (for each media category)
$\sum \mathrm{ad}_{\text {Aboite }} \sum \mathrm{ad}_{\text {Dupont }}$ does not exceed 3 for each quarter
$\sum \operatorname{ad}_{\text {magazines }}$ is at least 1 but no more than 3 each quarter
$\sum \mathrm{ad}_{\text {YellowBook }}$ and $\sum \mathrm{ad}_{\text {Superpages }}$ may only equal 0 or 1 for each quarter $\sum \operatorname{ad}_{\text {STVIN }}$ and $\sum \operatorname{ad}_{\text {LADY }}$ must each be no greater than 1 and $\sum \operatorname{ad}_{\text {STVIN }}$ must $=\sum \mathrm{ad}_{\text {LADY }}$ $\sum \mathrm{ad}_{\text {PHB1 }}$ and $\sum \operatorname{ad}_{\text {PHB2 }}$ must each be no greater than 1 and $\sum \operatorname{ad}_{\text {PHBI }}$ must $=\sum \operatorname{ad}_{\text {PHB2 }}$ $\sum \operatorname{ad}_{\text {PHB3 }}$ and $\sum \operatorname{ad}_{\text {PHB4 }}$ must each be no greater than 1 and $\sum \operatorname{ad}_{\text {PHB3 }}$ must $=\sum \operatorname{ad}_{\text {PHB4 }}$

## Equation Model

Since this model is designed to capture two intents in distinct priorities, it is submitted to the optimization software (LINDO) sequentially by priority level.

First Priority Goal: The first priority intent is to minimize the overachievement deviation of the budget constraints.
i.e. MIN d1plus + d2plus + d3plus + d4plus $+2 d 5$ plus
subject to: \{for the complete constraint set of 72 equations, see Appendix B \}
Once that optimal solution has been established, the focus shifts to the second priority goal.

Second Priority Goal: The second goal is to maximize annual exposures (while maintaining the optimal value of the first goal) was achieved with the following objective function.
MAX
170070WAJI1 + 170070WAJI2 + 170070WAJI3 + 170070WAJI4 + 181914WLDE1 + 181914WLDE $2+181914$ WLDE3 + 181914WLDE $4+290400$ FWND $1+$ 290400FWND 2 + 290400FWND3 + 290400FWND $4+842700$ FWNS $1+$ $842700 \mathrm{FWNS} 2+842700 \mathrm{FWNS} 3+842700 \mathrm{FWNS} 4+58400 \mathrm{ABNE} 1+58400 \mathrm{ABNE} 2+$ $58400 \mathrm{ABNE} 3+58400 \mathrm{ABNE} 4+57000 \mathrm{TIMDUP} 1+57000 \mathrm{TIMDUP} 2+$ 57000TIMDUP3 + 57000TIMDUP4 + 97158FWNM1 + 97158FWNM2 + 97158FWNM3 + 97158FWNM4 + 156000NILM1 + 156000NILM $2+156000$ NILM3 + 156000NILM4 + 195900BUWM1 + 195900BUWM2 + 195900BUWM3 + 195900BUWM4 + 12000MDNM1 + 12000MDNM 2 + 12000MDNM3 + 12000MDNM4
$+60000 \mathrm{FWMON} 1+60000 \mathrm{FWMON} 2+60000 \mathrm{FWMON} 3+60000 \mathrm{FWMON} 4+$ 52800 FWPAR $1+52800$ FWPAR $2+52800$ FWPAR $3+52800$ FWPAR $4+$ 24000FWWEB $1+24000$ FWWEB $2+24000$ FWWEB $3+24000$ FWWEB $4+$ 669YELLOF1 + 669YELLOF2 + 669YELLOF3 + 669YELLOF4 + 1167SUPER1 + 1167 SUPER $2+1167$ SUPER $3+1167$ SUPER $4+3300$ STVIN $1+3300$ STVIN $2+$ 3300STVIN3 + 3300STVIN4 + 2400LADY1 + 2400LADY2 + 2400LADY3 + $2400 \mathrm{LADY} 4+6000 \mathrm{PHB} 1+4300 \mathrm{PHB} 2+4200 \mathrm{PHB} 3+5000 \mathrm{PHB} 4$
subject to: the same set of 72 constraints used in the first priority phase, plus a new restriction that enforces the optimal objective function value found in the first phase. This is accomplished via the inclusion of:
G1) d1plus + d2plus + d3plus + d4plus $+2 d 5$ plus $=0$

## Optimal Solution

After 1628 iterations and 89 branches through the "Branch and Bound" enforcement of the integer requirements, the multi-period advertising portfolio model produced the results summarized in Table 8 below. These values represent the recommended level of ads to purchase and the resulting deviations from the budgetary targets.

First Priority Goal: $\quad$ Objective Function $=0.0000$
Because the first goal only sought to minimize the overachievement of the budget deviations, the value of the objective function is the only important output to be drawn from this initial run. Moreover, we learn that a mix of ads satisfying the constrained environment, without budget overruns, is indeed possible. The next optimization run addressed exposures while enforcing the optimal value of the first priority objective function (i.e. zero cost overruns).

Second Priority Goal: $\quad$ Objective Function Value $=55,952,240$
Table 8: Optimal Portfolio of Advertisements

| Variable | \# of Ads | Variable | \# of Ads | Variable | \# of Ads |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Radio Ads |  | Local Web-Based Ads |  | Local Magazine Ads |  |
| WLDE1 | 89.000000 | FWWEB1 | 2.000000 | FWNM1 | 3.000000 |
| WLDE2 | 4.000000 | FWWEB2 | 5.000000 | FWNM4 | 1.000000 |
| WLDE3 | 79.000000 | YELLOF1 | 1.000000 | BUWM2 | 3.000000 |
| WLDE4 | 23.000000 | YELLOF3 | 1.000000 | BUWM3 | 3.000000 |
| Newspaper Ads |  | YELLOF4 | 1.000000 | Positive Deviation Variables |  |
| FWNS2 | 2.000000 | SUPER1 | 1.000000 | D1minus | 20.48 |
| FWNS4 | 19.000000 | SUPER1 | 1.000000 | D2minus | 27.32 |
| ABNE1 | 3.000000 | SUPER3 | 1.000000 | D3minus | 5.27 |
| ABNE4 | 3.000000 | SUPER4 | 1.000000 | D4minus | 60.06 |
| TIMDUP1 | 3.000000 | Local Philharmonic Ads |  | D5minus | 320.13 |
| TIMDUP2 | 3.000000 | PHB3 | 1.000000 |  |  |
| TIMDUP3 | 3.000000 | PHB4 | 1.000000 |  |  |
| TIMDUP4 | 3.000000 |  |  |  |  |

The resulting slack or surplus associated with each constraint is reported in Table 9 below; this enables the identification of binding constraints.

Table 9: Resulting Slack / Surplus for Each Constraint

| Constraint | Slack / <br> Surplus |
| :---: | :---: |
| BDG1 | 0.000000 |
| BGT2 | 0.000000 |
| BGT3 | 0.000000 |
| BGT4 | 0.000000 |
| BGTANN | 0.000000 |
| NEWBGT | 10.509975 |
| MAGBGT | 230.489883 |
| RADBGT | 0.360056 |
| INTBGT | 2963.010010 |
| CHUBGT | 7759.009766 |
| PHBBGT | 6495.009766 |
| ABNEAV1 | 0.000000 |
| ABNEAV2 | 3.000000 |
| ABNEAV3 | 3.000000 |
| ABNEAV4 | 0.000000 |
|  |  |
|  |  |


| Constraint | Slack / <br> Surplus |
| :---: | :---: |
| TIMDUPA1 | 0.000000 |
| TIMDUPA2 | 0.000000 |
| TIMDUPA3 | 0.000000 |
| TIMDUPA4 | 0.000000 |
| MAGMIN1 | 2.000000 |
| MAGMIN2 | 2.000000 |
| MAGMIN3 | 2.000000 |
| MAGMIN4 | 0.000000 |
| MAGMAX1 | 0.000000 |
| MAGMAX2 | 0.000000 |
| MAGAMX3 | 0.000000 |
| MAGAMX4 | 2.000000 |
| CHB1 | 0.000000 |
| CHB2 | 0.000000 |
| CHB3 | 0.000000 |
| CHB4 | 0.000000 |
|  |  |


| Constraint | Slack / <br> Surplus |
| :---: | :---: |
| RADREQ1 | 88.000000 |
| RADREQ2 | 3.000000 |
| RADREQ3 | 78.000000 |
| RADREQ4 | 22.000000 |
| NEWREQ1 | 5.000000 |
| NEWREQ2 | 4.000000 |
| NEWREQ3 | 2.000000 |
| NEWREQ4 | 24.000000 |
| INTREQ1 | 3.000000 |
| INTREQ2 | 5.000000 |
| INTREQ3 | 1.000000 |
| INTREQ4 | 1.000000 |
| PH1 | 2.000000 |
| PHS1 | 0.000000 |
| PH2 | 0.000000 |
| PHS2 | 0.000000 |
| G1 | 0.000000 |

The slack/surplus values reported above that have a value of 0.000000 are indicative of binding constraints. The slack/surplus values greater than 0.000000 are non-binding constraints. For example, row NEWBGT has a positive slack value of 10.509975 , meaning that the optimal solution mix leaves $\$ 10.509975$ in the newspaper budget's proportionate cap; consequently, this was not a binding issue.
The objective function value indicates $55,952,240$ is the greatest number of exposures per year that can be generated by the advertising plan. The objective of the campaign design was to maximize the total number of exposures per year while also minimizing overspending of the budget. Therefore, it is not surprising that it chose some of the media alternatives with the highest number of exposures per dollar spent. (See Table 10 below for illustration.) Not all of the variables in the basis follow that logic, but there were several media requirements and limitations that affected the outcome.

TABLE 10: Exposures Per Dollar

| Media | Total \# of <br> Exposures per <br> quarter | Cost (\$) per <br> Ad | Ratio <br> Exposures per <br> \$ |
| :---: | :---: | :---: | :---: |
| WAJI1,2,3,4 | 170070 | $\$ 37.50$ | 4535.200000 |
| *WLDE1,2,3,4 | 181914 | $\$ 39.87$ | 4562.678706 |
| FWND1,2,3,4 | 290400 | $\$ 148.75$ | 1952.268908 |
| *FWNS1,2,3,4 | 842700 | $\$ 214.10$ | 3936.011210 |
| *ABNE1,2,3,4 | 59400 | $\$ 254.00$ | 233.858268 |
| *TIMDUP1,2,3,4 | 57000 | $\$ 254.00$ | 224.409449 |
| *FWNM1,2,3,4 | 97158 | $\$ 348.03$ | 279.165589 |
| NILM1,2,3,4 | 156000 | $\$ 1,215.00$ | 128.395062 |
| *BUWM1,2,3,4 | 195900 | $\$ 1,000.00$ | 195.900000 |
| MDNM1,2,3,4 | 12000 | $\$ 630.00$ | 19.047619 |
| FWMON1,2,3,4 | 60000 | $\$ 1,185.00$ | 50.632911 |
| FWPAR1,2,3,4 | 52800 | $\$ 695.00$ | 75.971223 |
| *YELLOF1,2,3,4 | 669 | $\$ 40.00$ | 16.725000 |
| *SUPER1,2,3,4 | 1167 | $\$ 123.00$ | 9.487805 |
| *FWWEB1,2,3,4 | 24000 | $\$ 600.00$ | 40.000000 |
| STVIN1,2,3,4 | 3300 | $\$ 160.00$ | 20.625000 |
| LADY1,2,3,4 | 2400 | $\$ 287.00$ | 8.362369 |
| PHB1 | 6000 | $\$ 1,205.00$ | 4.979253 |
| PHB2 | 4300 | $\$ 860.00$ | 5.000000 |
| *PHB3 | 4200 | $\$ 860.00$ | 4.883721 |
| *PHB4 | 5000 | $\$ 420.00$ | 11.904762 |

* Variables in the basis

We note that the recommended portfolio includes some of each candidate media except for the church bulletins. The collection includes quarterly radio ads (all with the same station), quarterly newspaper ads, with a heavy emphasis on the fourth quarter, quarterly magazine ads, with a reduced emphasis on the fourth quarter, quarterly web-based ads, with emphasis on the first half of the year, and philharmonic ads placed in just books 3
and 4 (i.e. the 'half-season' option). This portfolio can be achieved without any budget overruns.

The included policy constraints did not require the church bulletins to be used; their absence in the optimal solution indicates that these ads do not receive a large amount of exposure relative to the cost of purchasing the ads. If the decision is made to continue to advertise in the church bulletins, for either personal or political reasons, it will be with the knowledge that it is not part of the recommended optimal mix.

The proposed media mix differs significantly from the existing mix. Currently, the physician has large amounts of advertising dollars invested in only one or two media at a time, and the mix changes frequently. This practice is indicative of a lack of experience with advertising and the vulnerable nature of the physician's office to impulsive decision making about local media. Also, driven by a fear that the return on investment will be low, long term commitments to any single advertising plan are rarely made. The substantial differences between the current plan and the recommended optimal mix are precisely the reason this study was necessary.

## Sensitivity Analysis

Sensitivity analysis tests to see how the changes made to the parameters of model affect the optimal solution. Sensitivity analysis for the advertising portfolio model seeks to answer the following specific questions:

1. How will a change in the number of exposures generated from WLDE radio ads during Quarter 4 (WLDE4) affect the optimal solution? (i.e. a change in one selected objective function coefficient)
2. How will a change in the total annual budget affect the optimal solution? (i.e. a change in one right-hand-side target value)

Because the model enforced integer values via 'Branch and Bound', a meaningful and automated sensitivity analysis could not be obtained. Instead, it is necessary to "revise \& resubmit" the model multiple times while varying one parameter in order to gauge its impact.

The first revision dealt with a change in a coefficient for the objective function. The optimal solution did not include any radio ads for WAJI for any quarter. The cost per ad for WAJI (\$37.50) was less than the cost per ad for WLDE (\$39.87), but WLDE delivered more exposures per dollar spent ( 4562.678606 Exp/\$) than WAJI provided (4535.2 Exp/\$), so it was not surprising that WLDE was chosen over WAJI. In order to determine how much the coefficient of WLDE4 affects the value of the objective function, it was changed and the model was resubmitted. In the first scenario, the number of exposures generated from WLDE4 is dropped from 181,914 to 90,000 . (This could be result of the radio station going off air about halfway through the fourth quarter; or perhaps the station does not play Christmas music and so their listenership drops during the month of December.) In the second scenario, the number of exposures generated from WLDE4 is increased to 190,000 . (Perhaps the station plays only Christmas music
during Quarter 4, increasing the number listeners.) The same constraints were used from the original model. The results are displayed in Table 11.

Table 11: Varying a Coefficient in the Objective Function

| Coefficient of <br> WLDE4 | Objective <br> Function Value | Difference from Optimal <br> Obj. Function Value |
| :---: | :---: | :---: |
| 90,000 | $55,903,420$ | lose 48,820 exposures |
| 181,914 | $55,952,240$ | 0 |
| 190,000 | $55,973,720$ | gain 21,480 exposures |

The results of Table 11 demonstrate a significant drop of 48,820 in the objective function value if WLDE were to lose listeners in Quarter 4. It also shows that an increase in this coefficient by 9,000 exposures for the quarter improves the objective function value by more than 21,000. The objective function value changes significantly in both scenarios, because the Basis also changes. The variables in the Basis when WLDE4 exposures drop to 90,000 , and the variables in the basis when WLDE4 exposures rise to 190,000 are shown in Table 12 below. When just the value changes in response to the coefficient changes, just the value is highlighted; when the composition of the Basis changes as well, both the variable name and the value are highlighted.

TABLE 12: Comparison of the BASIS as One Coefficient Varies

| Coeff of WLDE4 <br> = 90,000 |  |
| :---: | :---: |
| Variable | Value |
| *WAJI3 | 1.000000 |
| *WAJI4 | 20.000000 |
| *WLDE1 | 32.000000 |
| *WLDE2 | 18.000000 |
| *WLDE3 | 125.000000 |
| WLDE4 | 0.000000 |
| *FWNS1 | 2.000000 |
| FWNS2 | 0.000000 |
| FWNS3 | 0.000000 |
| *FWNS4 | 19.000000 |
| *ABNE1 | 3.000000 |
| *ABNE4 | 3.000000 |
| *TIMDUP1 | 3.000000 |
| *TIMDUP2 | 3.000000 |
| *TIMDUP3 | 3.000000 |
| *TIMDUP4 | 3.000000 |


| Coeff of WLDE4 $=\mathbf{1 8 1}, 914$ <br> (original case) |  |
| :---: | :---: |
| Variable | Value |
| WAJI3 | 0.000000 |
| WAJI4 | 0.000000 |
| *WLDE1 | 89.000000 |
| *WLDE2 | 4.000000 |
| *WLDE3 | 79.000000 |
| *WLDE4 | 23.000000 |
| FWNS1 | 0.000000 |
| *FWNS2 | 2.000000 |
| FWNS3 | 0.000000 |
| *FWNS4 | 19.000000 |
| *ABNE1 | 3.000000 |
| *ABNE4 | 3.000000 |
| *TIMDUP1 | 3.000000 |
| *TIMDUP2 | 3.000000 |
| *TIMDUP3 | 3.000000 |
| *TIMDUP4 | 3.000000 |


| Coeff of WLDE4 <br> $=190,000$ |  |
| :---: | :---: |
|  | Value |
| WAJI3 | 0.000000 |
| WAJI4 | 0.000000 |
| *WLDE1 | 69.000000 |
| *WLDE2 | 17.000000 |
| *WLDE3 | 74.000000 |
| *WLDE4 | 35.000000 |
| FWNS1 | 0.000000 |
| FWNS2 | 0.000000 |
| *FWNS3 | 2.000000 |
| *FWNS4 | 19.000000 |
| *ABNE1 | 3.000000 |
| ABNE4 | 0.000000 |
| *TIMDUP1 | 3.000000 |
| *TIMDUP2 | 3.000000 |
| *TIMDUP3 | 2.000000 |
| *TIMDUP4 | 3.000000 |


| FWNM1 | 0.000000 | *FWNM1 | 3.000000 | FWNM1 | 0.000000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| *FWNM3 | 2.000000 | FWNM3 | 0.000000 | *FWNM3 | 1.000000 |
| *FWNM4 | 2.000000 | *FWNM4 | 1.000000 | *FWNM4 | 2.000000 |
| *BUWM1 | 3.000000 | BUWM1 | 0.000000 | *BUWM1 | 2.000000 |
| *BUWM2 | 3.000000 | *BUWM2 | 3.000000 | *BUWM2 | 3.000000 |
| BUWM3 | 0.000000 | *BUWM3 | 3.000000 | *BUWM3 | 1.000000 |
| *FWWEB1 | 2.000000 | *FWWEB1 | 2.000000 | *FWWEB1 | 2.000000 |
| *FWWEB2 | 5.000000 | *FWWEB2 | 5.000000 | *FWWEB2 | 5.000000 |
| *FWWEB3 | 1.000000 | FWWEB3 | 0.000000 | FWWEB3 | 0.000000 |
| YELLOF1 | 0.000000 | *YELLOF1 | 1.000000 | YELLOF1 | 0.000000 |
| YELLOF2 | 0.000000 | YELLOF2 | 0.000000 | *YELLOF2 | 1.000000 |
| YELLOF3 | 0.000000 | *YELLOF3 | 1.000000 | *YELLOF3 | 1.000000 |
| *YELLOF4 | 1.000000 | *YELLOF4 | 1.000000 | YELLOF4 | 0.000000 |
| SUPER1 | 0.000000 | *SUPER1 | 1.000000 | SUPER1 | 0.000000 |
| SUPER2 | 0.000000 | *SUPER2 | 1.000000 | SUPER2 | 0.000000 |
| SUPER3 | 0.000000 | *SUPER3 | 1.000000 | SUPER3 | 0.000000 |
| SUPER4 | 0.000000 | *SUPER4 | 1.000000 | *SUPER4 | 1.000000 |
| *PHB3 | 1.000000 | *PHB3 | 1.000000 | *PHB3 | 1.000000 |
| *PHB4 | 1.000000 | *PHB4 | 1.000000 | *PHB4 | 1.000000 |
| D1MINUS | 71.960022 | D1MINUS | 20.480099 | D1MINUS | 24.970074 |
| D2MINUS | 20.340019 | D2MINUS | 27.319992 | D2MINUS | 20.210018 |
| D3MINUS | 0.690136 | D3MINUS | 5.270084 | D3MINUS | 5.390068 |
| D4MINUS | 2.039886 | D4MINUS | 60.059910 | D4MINUS | 35.589924 |
| D5MINUS | 95.030060 | D5MINUS | 320.130035 | D5MINUS | 401.160004 |

*Variables in the basis

We note that reducing the effective audience reached by WLDE has brought the second radio station into the Basis. Each permutation recommends that the local Sunday newspaper still takes precedence over the daily paper, but with quite different quarterly ads, save for the fourth quarter. The actual recommended set of magazine ads changes dramatically, yet the Lakes Magazine, the MD News Magazine and the local Parenting Magazine are never recommended. The mix of Internet ads is also dependent upon this chosen parameter. While all cases recommend the same use of the philharmonic ads, and all share an avoidance of use of the church bulletins, we do find considerable responsiveness to plausible changes in the exposures (objective function coefficient) from WLDE4.

Sensitivity analysis was also performed by varying the right hand side value for one of the constraints. Because the model was tightly controlled by the budget constraints, the value of the annual budget constraint was selected to be changed in order to observe its impact.

In the first permutation, the annual budget constraint was decreased from $\$ 30,000$ to $\$ 25,000$. The change decreased the objective function value to $48,132,230$ exposures per
year. The second permutation increased the annual budget constraint by $\$ 5,000$ to a new value of $\$ 35,000$. The change increased the objective function value to $55,970,238$ exposures per year. In each permutation, new variables entered the basis, creating a new media mix. These comparative results are in Table 13 below. As with the earlier analysis, when just the value changes in response to the parameter changes, just the value is highlighted; when the composition of the Basis changes as well, both the variable name and the value are highlighted.

TABLE 13: Comparison of the BASIS as One RHS Target Varies

| RHS of Annual Budget <br> = \$25,000 |  |
| :--- | :--- |
| Variable | Value |
| *WLDE1 | 1.000000 |
| *WLDE2 | 3.000000 |
| *WLDE3 | 5.000000 |
| *WLDE4 | 132.000000 |
| *FWNS1 | 6.000000 |
| *FWNS2 | 8.000000 |
| *FWNS3 | 8.000000 |
| *FWNS4 | 2.000000 |
| *ABNE1 | 3.000000 |
| ABNE4 | 0.000000 |
| *TIMDUP1 | 3.000000 |
| *TIMDUP2 | 3.000000 |
| *TIMDUP3 | 3.000000 |
| *TIMDUP4 | 3.000000 |
| *FWNM1 | 3.000000 |
| *FWNM2 | 2.000000 |
| *FWNM4 | 1.000000 |
| BUWM2 | 0.000000 |
| *BUWM3 | 3.000000 |
| *FWWEB1 | 6.000000 |
| *FWWEB2 | 2.000000 |
| *FWWEB3 | 1.000000 |
| YELLOF1 | 0.000000 |
| *YELLOF2 | 1.000000 |
| YELLOF3 | 0.000000 |
| *YELLOF4 | 1.000000 |
| SUPER1 | 0.000000 |
| SUPER2 | 0.000000 |
| SUPER3 | 0.000000 |
| *SUPER4 | 1.000000 |
| *PHB3 | 1.000000 |
| *PHB4 | 1.000000 |
| D1MINUS | 7.439968 |
|  |  |


| RHS of Annual Budget <br> = \$25,000 <br> (original case) |  |
| :--- | :--- |
| Variable | Value |
| *WLDE1 | 89.000000 |
| *WLDE2 | 4.000000 |
| *WLDE3 | 79.000000 |
| *WLDE4 | 23.000000 |
| FWNS1 | 0.000000 |
| *FWNS2 | 2.000000 |
| FWNS3 | 0.000000 |
| *FWNS4 | 19.000000 |
| *ABNE1 | 3.000000 |
| *ABNE4 | 3.000000 |
| *TIMDUP1 | 3.000000 |
| *TIMDUP2 | 3.000000 |
| *TIMDUP3 | 3.000000 |
| *TIMDUP4 | 3.000000 |
| *FWNM1 | 3.000000 |
| FWNM2 | 0.000000 |
| *FWNM4 | 1.000000 |
| *BUWM2 | 3.000000 |
| *BUWM3 | 3.000000 |
| *FWWEB1 | 2.000000 |
| *FWWEB2 | 5.000000 |
| FWWEB3 | 0.000000 |
| *YELLOF1 | 1.000000 |
| YELLOF2 | 0.000000 |
| *YELLOF3 | 1.000000 |
| *YELLOF4 | 1.000000 |
| *SUPER1 | 1.000000 |
| *SUPER2 | 1.000000 |
| *SUPER3 | 1.000000 |
| *SUPER4 | 1.000000 |
| *PHB3 | 1.000000 |
| *PHB4 | 1.000000 |
| D1MINUS | 20.480099 |
|  |  |


| RHS of Annual Budget <br> $=$ <br> $\mathbf{\$ 3 5 , 0 0 0}$ |  |
| :--- | :--- |
| Variable | Value |
| *WLDE1 | 117.000000 |
| *WLDE2 | 18.000000 |
| *WLDE3 | 42.000000 |
| *WLDE4 | 18.000000 |
| FWNS1 | 0.000000 |
| FWNS2 | 0.000000 |
| *FWNS3 | 2.000000 |
| *FWNS4 | 19.000000 |
| *ABNE1 | 3.000000 |
| *ABNE4 | 3.000000 |
| *TIMDUP1 | 3.000000 |
| *TIMDUP2 | 3.000000 |
| *TIMDUP3 | 3.000000 |
| *TIMDUP4 | 3.000000 |
| *FWNM1 | 2.000000 |
| FWNM2 | 0.000000 |
| *FWNM4 | 2.000000 |
| *BUWM2 | 3.000000 |
| *BUWM3 | 3.000000 |
| *FWWEB1 | 1.000000 |
| *FWWEB2 | 5.000000 |
| FWWEB3 | 0.000000 |
| YELLOF1 | 0.000000 |
| YELLOF2 | 0.000000 |
| YELLOF3 | 0.000000 |
| *YELLOF4 | 1.000000 |
| SUPER1 | 0.000000 |
| SUPER2 | 0.000000 |
| SUPER3 | 0.000000 |
| SUPER4 | 0.000000 |
| *PHB3 | 1.000000 |
| *PHB4 | 1.000000 |
| D1MINUS | 15.150127 |
|  |  |


| D2MINUS | 2969.530029 |
| :--- | :--- |
| D3MINUS | 805.849976 |
| D4MINUS | 115.930130 |
| D5MINUS | 86.749756 |


| D2MINUS | 27.319992 |
| :--- | :--- |
| D3MINUS | 5.270084 |
| D4MINUS | 60.059910 |
| D5MINUS | 320.130035 |


| D2MINUS | 20.340019 |
| :--- | :--- |
| D3MINUS | 15.260033 |
| D4MINUS | 34.379906 |
| D5MINUS | 5247.129883 |

* Variables in the basis

Each of the annual budgetary permutations shares certain commonalities with the original case, these include: 1) all propose only one radio station (WLDE) yet with quite different quarterly recommendations; 2) all use the Sunday newspaper rather than the daily paper; 3) use of neighborhood papers, local magazines, and philharmonic ads (and absence of church bulletin ads) show considerable stability; 4) the most responsive set of recommended ads appears to be the mix of those using the Internet.

In sum, by using a 'revise and resubmit' approach on selected parameters, we are able to formalize the impact on both the final recommended media mix and the resulting exposures that might be attributed to changes in such parameters in the design of the model. Sensitivity analysis enables the decision maker to make informed responses to changes in a dynamic world. While such post-optimality analysis has been done above only for two parameters of interest, it could be undertaken for any/all of the parameters around which the model is built.

## Conclusions and Directions for Future Research

The purpose of the advertising portfolio model was to discover the optimal mix of media alternatives to maximize exposures given budget constraints and media usage requirements and limitations. Goal programming was implemented by introducing deviation variables to the budget constraints to allow for flexibility in the solution. The solution produced from the model is meaningful and will guide the physician's office in purchasing advertising space in the various media sources. The media sources used for this model were all sources with which the physician was already familiar. Further research could be done to investigate other media alternatives available in the area, including television, email, and direct mail advertising.

It may also be worthwhile to re-discuss some of the constraints that require a minimum amount of a certain media source or that limit the use of a media source. Rather than diversify the mix in that manner, perhaps a quality rating system could be used, and only media that meet a specific quality requirement may be used.

Optimization modeling has seemingly endless potential for developing and solving models that can assist in making optimal business decisions. The health-care industry has only begun to link its relatively recent open attitude regarding advertising with optimization techniques that can assist the medical practitioner 'do the best it can with what it has to work with'. The 'prognosis is good' for a contributing role for advertising in this professional service industry.

## Appendix A: The Decision Variables

Table 1: Radio Alternatives

| Variable <br> Symbol | Variable Description | Variable Type |
| :--- | :--- | :--- |
| WAJI1 | Number of 60s radio ads on WAJI in Quarter 1 | Counting Integer |
| WAJI2 | Number of 60s radio ads on WAJI in Quarter 2 | Counting Integer |
| WAJI3 | Number of 60s radio ads on WAJI in Quarter 3 | Counting Integer |
| WAJI4 | Number of 60s radio ads on WAJI in Quarter 4 | Counting Integer |
| WLDE1 | Number of 60s radio ads on WLDE in Quarter 1 | Counting Integer |
| WLDE2 | Number of 60s radio ads on WLDE in Quarter 2 | Counting Integer |
| WLDE3 | Number of 60s radio ads on WLDE in Quarter 3 | Counting Integer |
| WLDE4 | Number of 60s radio ads on WLDE in Quarter 4 | Counting Integer |

Table 2: Newspaper Alternatives

| Symbol | Variable Description | Variable Type |
| :---: | :--- | :--- |
| FWND1 | Number of 2x5 ads in the daily newspaper <br> in Quarter 1 | Counting Integer |
| FWND2 | Number of 2x5 ads in the daily newspaper <br> in Quarter 2 | Counting Integer |
| FWND3 | Number of 2x5 ads in the daily newspaper <br> in Quarter 3 | Number of 2x5 ads in the daily newspaper <br> in Quarter 4 |
| FWND4 | Counting Integer |  |
| FWNS1 | Number of 2x5 ads in the Sunday newspaper <br> in Quarter 1 | Counting Integer |
| FWNS2 | Number of 2x5 ads in the Sunday newspaper <br> in Quarter 2 | Counting Integer |
| FWNS3 | Number of 2x5 ads in the Sunday newspaper <br> in Quarter 3 | Counting Integer |
| FWNS4 | Number of 2x5 ads in the Sunday newspaper <br> in Quarter 4 | Counting Integer |
| ABNE1 | Number of 1/4 page ads in Aboite \& About <br> newspaper in Quarter 1 | Counting Integer |
| ABNE2 | Number of 1/4 page ads in Aboite \& About <br> newspaper in Quarter 2 | Counting Integer |
| ABNE3 | Number of 1/4 page ads in Aboite \& About <br> newspaper in Quarter 3 | Counting Integer |
| ABNE4 | Number of 1/4 page ads in Aboite \& About <br> newspaper in Quarter 4 | Counting Integer |
| TIMDUP1 | Number of 1/4 page ads in Dupont Valley Times <br> newspaper in Quarter 1 | Counting Integer |
| TIMDUP2 | Number of 1/4 page ads in Dupont Valley Times <br> newspaper in Quarter 2 | Counting Integer |


| TIMDUP3 | Number of 1/4 page ads in Dupont Valley Times <br> newspaper in Quarter 3 | Counting Integer |
| :--- | :--- | :--- |
| TIMDUP4 | Number of 1/4 page ads in Dupont Valley Times <br> newspaper in Quarter 4 | Counting Integer |

Table 3: Magazine Alternatives

| Symbol | Variable Description | Variable Type |
| :---: | :---: | :---: |
| FWNM1 | Number of $1 / 2$ page ads in FW Newspapers Magazine in Quarter 1 | Counting Integer |
| FWNM2 | Number of $1 / 2$ page ads in FW Newspapers Magazine in Quarter 2 | Counting Integer |
| FWNM3 | Number of $1 / 2$ page ads in FW Newspapers Magazine in Quarter 3 | Counting Integer |
| FWNM4 | Number of $1 / 2$ page ads in FW Newspapers Magazine in Quarter 4 | Counting Integer |
| NILM1 | Number of $1 / 2$ page ads in Northern IN Lakes Magazine in Quarter 1 | Counting Integer |
| NILM2 | Number of $1 / 2$ page ads in Northern IN Lakes Magazine in Quarter 2 | Counting Integer |
| NILM3 | Number of $1 / 2$ page ads in Northern IN Lakes Magazine in Quarter 3 | Counting Integer |
| NILM4 | Number of $1 / 2$ page ads in Northern IN Lakes Magazine in Quarter 4 | Counting Integer |
| BUWM1 | Number of $1 / 3$ page ads in Business Women Magazine in Quarter 1 | Counting Integer |
| BUWM2 | Number of $1 / 3$ page ads in Business Women Magazine in Quarter 2 | Counting Integer |
| BUWM3 | Number of $1 / 3$ page ads in Business Women Magazine in Quarter 3 | Counting Integer |
| BUWM4 | Number of $1 / 3$ page ads in Business Women Magazine in Quarter 4 | Counting Integer |
| MDNM1 | Number of $1 / 4$ page ads in MD News Magazine in Quarter 1 | Counting Integer |
| MDNM2 | Number of $1 / 4$ page ads in MD News Magazine in Quarter 2 | Counting Integer |
| MDNM3 | Number of $1 / 4$ page ads in MD News Magazine in Quarter 3 | Counting Integer |
| MDNM4 | Number of $1 / 4$ page ads in MD News Magazine in Quarter 4 | Counting Integer |
| FWMON1 | Number of 1/4 page ads in FW Monthly Magazine in Quarter 1 | Counting Integer |
| FWMON2 | Number of $1 / 4$ page ads in FW Monthly Magazine in Quarter 2 | Counting Integer |
| FWMON3 | Number of $1 / 4$ page ads in FW Monthly Magazine in Quarter 3 | Counting Integer |


| FWMON4 | Number of 1/4 page ads in FW Monthly <br> Magazine in Quarter 4 | Counting Integer |
| :--- | :--- | :--- |
| FWPAR1 | Number of 1/4 page ads in FW Parenting <br> Magazine in Quarter 1 | Counting Integer |
| FWPAR2 | Number of 1/4 page ads in FW Parenting <br> Magazine in Quarter 2 | Counting Integer |
| FWPAR3 | Number of 1/4 page ads in FW Parenting <br> Magazine in Quarter 3 | Counting Integer |
| FWPAR4 | Number of 1/4 page ads in FW Parenting <br> Magazine in Quarter 4 | Counting Integer |

Table 4: Internet Alternative

| Symbol | Variable Description | Variable Type |
| :---: | :--- | :--- |
| FWWEB1 | Number 160x9 ads on FortWayne.com <br> in Quarter 1 | Counting Integer |
| FWWEB2 | Number 160x9 ads on FortWayne.com <br> in Quarter 2 | Counting Integer |
| FWWEB3 | Number 160x9 ads on FortWayne.com <br> in Quarter 3 | Number 160x9 ads on FortWayne.com <br> in Quarter 4 |
| FWWEB4 | Counting Integer |  |
| YELLOF1 | Number of website listings on <br> YellowBook.com in Quarter 1 | Counting Integer |
| YELLOF2 | Number of website listings on <br> YellowBook.com in Quarter 2 | Binary Integer |
| YELLOF3 | Number of website listings on <br> YellowBook.com in Quarter 3 | Binary Integer |
| YELLOF4 | Number of website listings on <br> YellowBook.com in Quarter 4 | Binary Integer |
| SUPER1 | Number of website listings on <br> Superpages.com in Quarter 1 | Binary Integer |
| SUPER2 | Number of website listings on <br> Superpages.com in Quarter 2 | Binary Integer |
| SUPER3 | Number of website listings on <br> Superpages.com in Quarter 3 | Binary Integer |
| SUPER4 | Number of website listings on <br> Superpages.com in Quarter 4 | Binary Integer |

Table 5: Church Bulletin Alternatives

| Symbol | Variable Description | Variable Type |
| :--- | :--- | :--- |
| STVIN1 | Number of 3x3 ads in St. Vincent Catholic <br> Church bulletin in Quarter 1 | Binary Integer |
| STVIN2 | Number of 3x3 ads in St. Vincent Catholic <br> Church bulletin in Quarter 2 | Binary Integer |
| STVIN3 | Number of 3x3 ads in St. Vincent Catholic <br> Church bulletin in Quarter 3 | Binary Integer |
| STVIN4 | Number of 3x3 ads in St. Vincent Catholic <br> Church bulletin in Quarter 4 | Binary Integer |
| LADY1 | Number of 3x3 ads in Our Lady of Good Hope <br> Catholic Church bulletin in Quarter 1 | Binary Integer |
| LADY2 | Number of 3x3 ads in Our Lady of Good Hope <br> Catholic Church bulletin in Quarter 2 | Binary Integer |
| LADY3 | Number of 3x3 ads in Our Lady of Good Hope <br> Catholic Church bulletin in Quarter 3 | Binary Integer |
| LADY4 | Number of 3x3 ads in Our Lady of Good Hope <br> Catholic Church bulletin in Quarter 4 | Binary Integer |

Table 6: Philharmonic Book Alternatives

| Symbol | Variable Description | Variable Type |
| :--- | :--- | :--- |
| PHB1 | Number of 1/3 page ads in FW Philharmonic <br> Book 1 (Quarter 1) | Binary Integer |
| PHB2 | Number of 1/3 page ads in FW Philharmonic <br> Book 2 (Quarter 2) | Binary Integer |
| PHB3 | Number of 1/3 page ads in FW Philharmonic <br> Book 3 (Quarter 3) | Binary Integer |
| PHB4 | Number of 1/3 page ads in FW Philharmonic <br> Book 4 (Quarter 4) | Binary Integer |

## Appendix B: The Full Equation Model

## First Goal:

The following is the full equation model for the first goal to minimize the overachievement deviation of the budget constraints.
MIN d1plus + d2plus + d3plus + d4plus +2 d5plus
ST
BDG1) 37.50WAJI1 + 39.87WLDE1 + 148.75FWND1 + 214.10FWNS1 + 254ABNE1 + 254 TIMDUP $1+348.03$ FWNM $1+1215$ NILM $1+630$ MDNM $1+1000$ BUWM $1+$ 1185FWMON $1+695$ FWPAR $1+40$ YELLOF $1+123$ SUPER $1+600$ FWWEB $1+$ 160STVIN1 + 287LADY1 + 1205PHB1 + d1minus - d1plus $=7500$
BGT2) 37.50WAJI2 + 39.87WLDE2 + 148.75FWND2 + 214.10FWNS2 + 254ABNE2 + 254 TIMDUP $2+348.03$ FWNM $2+1215$ NILM $2+630$ MDNM $2+1000$ BUWM $2+$ 1185 FWMON $2+695$ FWPAR $2+40$ YELLOF $2+123$ SUPER $2+600$ FWWEB $2+$ 160STVIN $2+287$ LADY $2+860$ PHB $2+$ d2minus - d2plus $=7500$

BGT3) 37.50WAJI3 + 39.87WLDE3 + 148.75FWND3 + 214.10FWNS3 + 254ABNE3 + 254 TIMDUP3 + 348.03FWNM3 + 1215NILM3 + 630MDNM3 + 1000BUWM3 + 1185FWMON3 + 695FWPAR3 + 40YELLOF3 + 123SUPER3 + 600FWWEB3 + 160STVIN3 + 287LADY3 +420 PHB3 + d3minus - d3plus $=7500$ BGT4) 37.50WAJI4 + 39.87WLDE4 + 148.75FWND4 + 214.10FWNS4 + 254ABNE4 + 254TIMDUP4 + 348.03FWNM4 + 1215NILM4 + 630MDNM4 + 1000BUWM4 + 1185FWMON4 + 695FWPAR4 + 40YELLOF4 + 123SUPER4 + 600FWWEB4 + 160STVIN4 + 287LADY4 + 420PHB4 + d4minus - d4plus $=7500$ BGTANN) $37.50 \mathrm{WAJI} 1+37.50 \mathrm{WAJI} 2+37.50 \mathrm{WAJI} 3+37.50 \mathrm{WAJI} 4+39.87 \mathrm{WLDE} 1$ + 39.87WLDE 2 + 39.87WLDE3 + 30.87WLDE 4 + 148.75FWND 1 + 148.75 FWND $2+$ 148.75FWND3 + 148.75FWND 4 + 214.10FWNS 1 + 214.10FWNS $2+214.10$ FWNS3 + 214.10FWNS4 + 254ABNE1 + 254ABNE2 + 254ABNE3 + 254ABNE4 + 254 TIMDUP $1+254$ TIMDUP 2 + 254TIMDUP3 +254 TIMDUP $4+348.03$ FWNM $1+$ 348.03 FWNM $2+348.03$ FWNM $3+348.03$ FWNM $4+1215$ NILM $1+1215$ NILM $2+$ 1215NILM3 + 1215NILM4 + 630MDNM1 + 630MDNM2 + 630MDNM3 + 630MDNM4 + 1000BUWM1 + 1000BUWM2 + 1000BUWM3 + 1000BUWM4 + 1185FWMON1 + 1185FWMON2 + 1185FWMON3 + 1185FWMON4 + 695FWPAR1 + 695FWPAR $2+695$ FWPAR $3+695$ FWPAR $4+40$ YELLOF $1+40$ YELLOF $2+$ 40YELLOF3 + 40YELLOF4 + 123SUPER1 + 123SUPER2 + 123SUPER3 + 123 SUPER $4+600$ FWWEB $1+600$ FWWEB $2+600$ FWWEB $3+600$ FWWEB 4 +160STVIN1 + 160STVIN2 + 160STVIN3 + 160STVIN4 + 287LADY1 + 287LADY2 $+287 \mathrm{LADY} 3+287 \mathrm{LADY} 4+1205 \mathrm{PHB} 1+860 \mathrm{PHB} 2+420 \mathrm{PHB} 3+420 \mathrm{PHB} 4+$ d5minus - d5plus $=30000$
NEWBGT) 119FWND1 + 119FWND2 + 119FWND3 + 119FWND4 + 171.28FWNS1 + 171.28FWNS2 + 171.28FWNS3 + 171.28FWNS4-7.5WAJI1 - 7.5WAJI2 - 7.5WAJI3 7.5WAJI4 - 7.97WLDE1-7.97WLDE2 - 7.97WLDE3 - 7.97WLDE4 - 82FWNS4 50.8ABNE1 50.8ABNE2 50.8ABNE3 - 50.8ABNE4 - 50.8TIMDUP1 - 50.8TIMDUP2 - 50.8TIMDUP3 - 50.8TIMDUP4-69.61FWNM1 - 69.61FWNM2 - 69.61FWNM3 69.61FWNM4 - 243NILM1-243NILM2-243NILM3-243NILM4-126MDNM1 -126MDNM2-126MDNM3 - 126MDNM4 - 200BUWM1 - 200BUWM2-200BUWM3 - 200BUWM4 - 237FWMON1-237FWMON2-237FWMON3-237FWMON4 -139FWPAR1-139FWPAR2-139FWPAR3-139FWPAR4-8YELLOF1-8YELLOF2 - 8YELLOF3 - 8YELLOF4 - 24.6SUPER1 - 24.6SUPER2 - 24.6SUPER3 -24.6SUPER4-120FWWEB1-120FWWEB2-120FWWEB3-120FWWEB4 32STVIN1 - 32STVIN2 - 32STVIN3 - 32STVIN4 - 57.4LADY1 - 57.4LADY2 -57.4LADY3-57.4LADY4-241PHB1-172PHB2-172PHB3-84PHB4<=0 MAGBGT) 278.42FWNM1 + 278.42FWNM2 + 278.42FWNM3 + 278.42FWNM4 + $972 \mathrm{NILM} 1+972 \mathrm{NILM} 2+972 \mathrm{NILM} 3+972 \mathrm{NILM} 4+504 \mathrm{MDNM} 1+504 \mathrm{MDNM} 2+$ 504MDNM3 + 504MDNM4 + 800BUWM1 + 800BUWM2 + 800BUWM3 + 800BUWM4 + 948FWMON1 + 948FWMON2 + 948FWMON3 + 948FWMON4 + 556FWPAR1 + 556FWPAR2 + 556FWPAR3 + 556FWPAR4-7.5WAJI1 - 7.5WAJI2 7.5WAJI3 - 7.5WAJI4 - 7.97WLDE1 - 7.97WLDE2 - 7.97WLDE3 - 7.97WLDE4 29.75FWND1 - 29.75FWND2 - 29.75FWND3 - 29.75FWND4 - 42.82FWNS1 42.82FWNS2 - 42.82FWNS3 - 42.82FWNS4 - 82FWNS4 - 50.8ABNE1 50.8ABNE2 50.8ABNE3 - 50.8ABNE4 - 50.8TIMDUP1 - 50.8TIMDUP2 - 50.8TIMDUP3 -50.8TIMDUP4-8YELLOF1-8YELLOF2 - 8YELLOF3 - 8YELLOF4 - 24.6SUPER1 -
24.6SUPER2 - 24.6SUPER3 - 24.6SUPER4-120FWWEB1-120FWWEB2 -120FWWEB3-120FWWEB4-32STVIN1 - 32STVIN2 - 32STVIN3-32STVIN4 57.4LADY1 - 57.4LADY2 - 57.4LADY3 - 57.4LADY4-241PHB1 - 172PHB2 -172PHB3-84PHB4 <= 0
RADBGT) 30WAJI1 + 30WAJI2 + 30WAJI3 + 30WAJI4 + 31.9WLDE1 + 31.9WLDE2 + 31.9WLDE3 + 31.9WLDE4 - 29.75FWND1 - 29.75FWND2 - 29.75FWND3 29.75FWND4 - 42.82FWNS1 - 42.82FWNS2 - 42.82FWNS3 - 42.82FWNS4 82FWNS4 - 50.8ABNE1 50.8ABNE2 50.8ABNE3 - 50.8ABNE4 - 50.8TIMDUP1 50.8TIMDUP2 - 50.8TIMDUP3-50.8TIMDUP4-50.8TIMDUP4-69.61FWNM1 69.61FWNM2 - 69.61FWNM3 - 69.61FWNM4 - 243NILM1-243NILM2-243NILM3 - 243NILM4-126MDNM1-126MDNM2-126MDNM3-126MDNM4 - 200BUWM1 -200BUWM2-200BUWM3 - 200BUWM4-237FWMON1-237FWMON2 -237FWMON3-237FWMON4-139FWPAR1-139FWPAR2-139FWPAR3-139FWPAR4-8YELLOF1-8YELLOF2 - 8YELLOF3-8YELLOF4 - 24.6SUPER1 -24.6SUPER2-24.6SUPER3-24.6SUPER4-120FWWEB1-120FWWEB2 -120FWWEB3-120FWWEB4-32STVIN1 - 32STVIN2 - 32STVIN3-32STVIN4 57.4LADY1 - 57.4LADY2 - 57.4LADY3 - 57.4LADY4 - 241PHB1 - 172PHB2 -172PHB3-84PHB4 <= 0
INTBGT) 32YELLOF1 + 32YELLOF2 + 32YELLOF3 + 32YELLOF4 + 98.4SUPER1
+98.4 SUPER $2+98.4$ SUPER $3+98.4$ SUPER $4+480$ FWWEB $1+480$ FWWEB $2+$ 480FWWEB3 + 480FWWEB4 - 7.5WAJI1 - 7.5WAJI2 - 7.5WAJI3-7.5WAJI4 7.97WLDE1 - 7.97WLDE2-7.97WLDE3-7.97WLDE4 - 29.75FWND1 29.75FWND2 - 29.75FWND3 - 29.75FWND4 - 42.82FWNS1 - 42.82FWNS2 42.82FWNS3 - 42.82FWNS4 - 82FWNS4 - 50.8ABNE1 50.8ABNE2 50.8ABNE3 50.8ABNE4 - 50.8TIMDUP1 - 50.8TIMDUP2 - 50.8TIMDUP3 - 50.8TIMDUP4 -50.8TIMDUP4-69.61FWNM1 - 69.61FWNM2 - 69.61FWNM3 - 69.61FWNM4 243NILM1 - 243NILM2-243NILM3-243NILM4 - 126MDNM1 - 126MDNM2 -126MDNM3-126MDNM4-200BUWM1-200BUWM2-200BUWM3-200BUWM4 - 237FWMON1-237FWMON2-237FWMON3-237FWMON4-139FWPAR1 -139FWPAR2-139FWPAR3-139FWPAR4-32STVIN1-32STVIN2-32STVIN3 32STVIN4 - 57.4LADY1 - 57.4LADY2 - 57.4LADY3 - 57.4LADY4 - 241PHB1 -172PHB2-172PHB3-84PHB4 <= 0 CHUBGT) 128STVIN1 + 128STVIN2 + 128STVIN3 + 128STVIN4 + 229.6LADY1 + 229.6LADY2 + 229.6LADY3 + 229.6LADY4 - 7.5WAJI1 - 7.5WAJI2 - 7.5WAJI3 7.5WAJI4 - 7.97WLDE1 - 7.97WLDE2 - 7.97WLDE3-7.97WLDE4 - 29.75FWND1 29.75FWND2 - 29.75FWND3 - 29.75FWND4 - 42.82FWNS1 - 42.82FWNS2 42.82FWNS3 - 42.82FWNS4 - 82FWNS4 - 50.8ABNE1 50.8ABNE2 50.8ABNE3 50.8ABNE4 - 50.8TIMDUP1 - 50.8TIMDUP2 - 50.8TIMDUP3 - 50.8TIMDUP4 -50.8TIMDUP4-69.61FWNM1 - 69.61FWNM2 - 69.61FWNM3 - 69.61FWNM4 243NILM1 - 243NILM2 - 243NILM3-243NILM4 - 126MDNM1 - 126MDNM2 -126MDNM3-126MDNM4-200BUWM1-200BUWM2-200BUWM3-200BUWM4 - 237FWMON1-237FWMON2-237FWMON3-237FWMON4-139FWPAR1 -139FWPAR2-139FWPAR3-139FWPAR4-139FWPAR1-139FWPAR2 -139FWPAR3-139FWPAR4-8YELLOF1-8YELLOF2-8YELLOF3-8YELLOF4 24.6SUPER1 - 24.6SUPER2 - 24.6SUPER3-24.6SUPER4 - 120FWWEB1 -

120FWWEB2-120FWWEB3-120FWWEB4-241PHB1-172PHB2-172PHB3-
84PHB4 <= 0
PHBBGT) 964PHB1 + 688PHB2 + 688PHB3 + 336PHB4 - 7.5WAJI1 - 7.5WAJI2 7.5WAJI3 - 7.5WAJI4 - 7.97WLDE1 - 7.97WLDE2 - 7.97WLDE3 - 7.97WLDE4 29.75FWND1 - 29.75FWND2 - 29.75FWND3 - 29.75FWND4 - 42.82FWNS1 42.82FWNS2 - 42.82FWNS3 - 42.82FWNS4 - 82FWNS4 - 50.8ABNE1 50.8ABNE2 50.8ABNE3 - 50.8ABNE4 - 50.8TIMDUP1 - 50.8TIMDUP2 - 50.8TIMDUP3 -
50.8TIMDUP4-50.8TIMDUP4-69.61FWNM1 - 69.61FWNM2 - 69.61FWNM3 69.61FWNM4 - 243NILM1-243NILM2-243NILM3-243NILM4-126MDNM1 -

126MDNM2-126MDNM3-126MDNM4-200BUWM1-200BUWM2-200BUWM3 - 200BUWM4-237FWMON1-237FWMON2-237FWMON3 - 237FWMON4 -

139FWPAR1-139FWPAR2-139FWPAR3-139FWPAR4-139FWPAR1 -
139FWPAR2-139FWPAR3-139FWPAR4-8YELLOF1-8YELLOF2-8YELLOF3 -
8YELLOF4 - 24.6SUPER1 - 24.6SUPER2 - 24.6SUPER3 - 24.6SUPER4 -
120FWWEB1-120FWWEB2 - 120FWWEB3 - 120FWWEB4-32STVIN1-32STVIN2 - 32STVIN3 - 32STVIN4 - 57.4LADY1 - 57.4LADY2 - 57.4LADY3 - 57.4LADY4 <= 0
ABNEAV1) ABNE1 <= 3
ABNEAV2) ABNE2 <= 3
ABNEAV3) ABNE3 <= 3
ABNEAV4) ABNE4 <= 3
TIMDUPA1) TIMDUP1 <= 3
TIMDUPA2) TIMDUP2 $<=3$
TIMDUPA3) TIMDUP3 <= 3
TIMDUPA4) TIMDUP4 <= 3
MAGMIN1) FWNM1 + NILM1 + BUWM1 + MDNM1 + FWMON1 + FWPAR1 >= 1
MAGMIN2) FWNM2 + NILM2 + BUWM2 + MDNM2 + FWMON2 + FWPAR2 >= 1
MAGMIN3) FWNM3 + NILM3 + BUWM3 + MDNM3 + FWMON3 + FWPAR3 >= 1
MAGMIN4) FWNM4 + NILM4 + BUWM4 + MDNM4 + FWMON4 + FWPAR4 >= 1
MAGMAX1) FWNM1 + NILM1 + BUWM1 + MDNM1 + FWMON1 + FWPAR1 <= 3
MAGMAX2) FWNM2 + NILM2 + BUWM2 + MDNM2 + FWMON2 + FWPAR2 <= 3
MAGMAX3) FWNM3 + NILM3 + BUWM3 + MDNM3 + FWMON3 + FWPAR3 <= 3
MAGMAX4) FWNM4 + NILM4 + BUWM4 + MDNM4 + FWMON4 + FWPAR4 <= 3
CHB1) STVIN1 - LADY1 $=0$
CHB2) STVIN2 - LADY2 $=0$
CHB3) STVIN3 - LADY3 $=0$
CHB4) STVIN4 - LADY4 $=0$
RADREQ1) WAJI1 + WLDE1 >= 1
RADREQ2) WAJI2 + WLDE2 >= 1
RADREQ3) WAJI3 + WLDE3 >= 1
RADREQ4) WAJI4 + WLDE4 >= 1
NEWREQ1) FWND1 + FWNS1 + ABNE1 + TIMDUP1 >= 1
NEWREQ2) FWND2 + FWNS2 + ABNE $2+$ TIMDUP2 $>=1$
NEWREQ3) FWND3 + FWNS3 + ABNE3 + TIMDUP3 >= 1
NEWREQ4) FWND4 + FWNS4 + ABNE4 + TIMDUP4 >= 1
INTREQ1) YELLOF1 + SUPER1 + FWWEB1 >= 1

```
INTREQ2) YELLOF2 + SUPER2 + FWWEB2 >= 1
INTREQ3) YELLOF3 + SUPER3 + FWWEB3 >= 1
INTREQ4) YELLOF4 + SUPER4 + FWWEB4 >= 1
PH1) PHB1 + PHB2 < = 2
PHS1) PHB1 - PHB2 \(=0\)
PH2) PHB3 + PHB4 <= 2
PHS2) PHB3 - PHB4 \(=0\)
END
```

Second Goal:
The second goal (to maximize annual exposures while maintaining the first goal) was achieved with the following objective function.

```
MAX
170070WAJI1 + 170070WAJI2 + 170070WAJI3 + 170070WAJI4 + 181914WLDE1 +
181914WLDE2 + 181914WLDE3 + 181914WLDE4 + 290400FWND1 +
290400FWND2 + 290400FWND3 + 290400FWND4 + 842700FWNS1 +
842700FWNS2 + 842700FWNS3 + 842700FWNS4 + 58400ABNE1 + 58400ABNE2 +
58400ABNE3 + 58400ABNE4 + 57000TIMDUP1 + 57000TIMDUP2 +
57000TIMDUP3 + 57000TIMDUP4 + 97158FWNM1 + 97158FWNM2 +
97158FWNM3 + 97158FWNM4 + 156000NILM1 + 156000NILM2 + 156000NILM3 +
156000NILM4 + 195900BUWM1 + 195900BUWM2 + 195900BUWM3 +
195900BUWM4 + 12000MDNM1 + 12000MDNM2 + 12000MDNM3 + 12000MDNM4
+60000FWMON1 + 60000FWMON2 + 60000FWMON3 + 60000FWMON4 +
52800FWPAR1 + 52800FWPAR2 + 52800FWPAR3 + 52800FWPAR4 +
24000FWWEB1 + 24000FWWEB2 + 24000FWWEB3 + 24000FWWEB4 +
669YELLOF1 + 669YELLOF2 + 669YELLOF3 + 669YELLOF4 + 1167SUPER1 +
1167SUPER2 + 1167SUPER3 + 1167SUPER4 + 3300STVIN1 + 3300STVIN2 +
3300STVIN3 + 3300STVIN4 + 2400LADY1 + 2400LADY2 + 2400LADY3 +
2400LADY4 + 6000PHB1 + 4300PHB2 + 4200PHB3 + 5000PHB4
```

ST
BDG1)...


PHS2) PHB3 - PHB4 = 0
G1) d1plus + d2plus + d3plus + d4plus $+2 d 5$ plus $=0$
END

## References

[1] American Medical Association v. FTC, 1980-1982. Trade Case 63, 569 (2 $2^{\text {nd }} \mathrm{Cir}$ 1980), Rehearing Denied, (November 25, 1980)
[2] Bates v. State Bar of Arizona (1977), 97 S.Ct. 2691, 2699, 2700.
[3] Carabello, L. (2003), "Quality of Information in Physician Advertising Under Close Scrutiny", Healthleader, June, 1-4.
[4] Darling, J.R. and R.E. Taylor, (1987), "A Longitudinal Comparative Study of the Changing Attitudes of Professionals Toward Advertising"; paper presented at the American Marketing Association Summer Educator's Conference, Toronto.
[5] Freedman, M., (2001), "New Techniques in Ambulance Chasing", Forbes, November, 56.
[6] Goldfarb v. Virginia State Bar Association (1975), 421 U.S. 773.
[7] Kluyver, C. (1978). Hard and Soft Constraints in Media Scheduling. Journal of Advertising Research, 18(3), 27-31.
[8] Lofgren, D., S. Rhodes, T. Miller, and J. Solomon, (2007), "Marketing the Health Care Experience: Eight Steps to Infuse Brand Essence into Your Organization", Health Marketing Quarterly, 23(3), 10.
[9] Marks, R. and S. Moon, (1994), "Comparative Analysis of Professional Advertising and Its Managerial Implications", Journal of Professional Services Marketing, 11(1), 127-142.
[10] Moser, R., (2008a). "An Empirical Analysis of Consumers’ Attitudes Toward Physicians’ Advertising", Health Marketing Quarterly, 25(3), 270-288.
[11] Moser, R., (2008b). "An Empirical Analysis of Consumers" Attitudes Toward Physicians' Services Advertising: A Comparative Cross-Sectional Study", Services Marketing Quarterly, 29(3), 75.
[12] Rifai, A. and N. Hanna, (2001) Planning the Media Mix Through Goal Programming, 21-26.
[13] Rizzo, J., (1988), Physician Advertising Revisited, Medical Care, 26 (Dec.), 12381244.
[14] Sahl, J., (2003), "The Cost of Humanitarian Assistance: Ethical Rules and the First Amendment", St. Mary's Law Journal, 34, 795.
[15] Sanchez, M. and P. Sanchez, (2006), "What Consumers Want from Physician Web Sites: An Exploratory Analysis", Services Marketing Quarterly, 28(2), 25.
[16] Virginia State Board of Pharmacy v. Virginia Citizens Consumer Council (1976), 425 U.S. 748; 96 S. Ct. 1817. Professionals' Use of Advertisin

## Biographies



Amy Boyer is an MBA candidate at Doermer School of Business and Management Sciences at Indiana University - Purdue University Fort Wayne, and the recipient of the Academy of Business Disciplines Best Student-Faculty Collaboration Award, 2009. She is the Billing Manager and Marketing Director of a private plastic surgery practice.


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