Modeling Nonlinear Pick Rates in the Mini-Pick Process at an IBM Replenishment Plant

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Abstract

This research focuses on constructing a modeling approach for a small volume warehouse pick operation with a nonlinear pick rate. Nonlinear pick rates exhibit a decreasing pick time per order for a specific line item when multiple orders are picked from one retrieval of or visit to a line item's bin or compartment. We construct a math programming model that would determine a pick schedule that has the lowest possible aggregate pick times while satisfying order due dates for all line items. This enables a pick schedule to be determined that minimizes pick costs while satisfying the pick due dates.

Introduction

The IBM Replenishment Plant in Silver Spring Township PA replenishes all IBM parts in North and South America, Asia, Australia, and the Far East. Sixty percent of their orders are same day shipping to customers or service technicians and forty percent of their orders are batch orders that are determined overnight as replenishment parts to other IBM Distribution Centers.

IBM has a special pick operation for the small replenishment orders that is called the Mini-Pick Operation. Depending upon where around the world the orders are being shipped, each order has a given time that it needs to be picked by. The Mini-Pick Operation uses a Mini-Load AS/RS System which is a mini-crane Automated Storage and Retrieval Material Handling System to pick orders. The crane retrieves a bin with the correct part number required for an order and once the part is removed and put on the order the crane replaces the bin.

Each order needs to be picked by a specific time during the day so that the order can be shipped to the customer on time. These pick times are determined by the location of the customer and the shipment mode and carrier(s). Thus, a part number may be required on numerous orders at different times throughout the day. Ideally the bin containing the part number will be retrieved only once that day and all of the orders that require that part number will be picked at that time. However, this is not always feasible due to labor requirements. Each time that part number is picked for an order, an order picker needs to remove the required number of parts from the bin, place the part(s) in the shipping package, and process the order on the Information System. If all orders are satisfied with one pick, the labor time may become so large that other part numbers for other orders may not be picked by the requisite time.

Let us consider an elementary two part example to illustrate this situation. Let us assume the following:

- Part 001 is needed on 20 orders
- Part 001 has three orders that need to be picked by 9:00
- Part 002 also is needed on 20 orders
- Part 002 also has three orders that need to be picked by 9:00
- The work day starts at 8:00 am
- The labor time spent on each part for each order is a constant 5 minutes
- There is one order picker

If all 20 orders of part 001 are picked in one AS/RS retrieval at 8:00 am, the labor time for all 20 orders is 100 minutes. Thus, the last of the 20 orders will not be picked and processed until at least 9:40 am. The first AS/RS retrieval for part 002 will not occur until at least 9:35 am so the three orders of part 002 due at 9:00 am will not be picked on time. The same argument applies if all 20 orders of part 002 are picked in one AS/RS retrieval at 8:00 am.

Productivity in the Mini-Pick Operation is measured in lines per hour with a line being a part number on an order. The time spent on an order is determined by the mini-crane time plus the labor time to remove the part from the bin, place the part in a ship package, and process the order on their Information System. The labor time is constant, but when more than one line is retrieved in one mini-pick then the mini-pick time can be split up among all of the lines picked. This reduces the total time spent to pick each of the lines and therefore increases productivity!

IBM would like to achieve the highest productivity possible while still picking all orders on-time throughout the day. An efficient scheduling method that groups multiple lines per pick when possible yet still maintains on-time delivery would enable IBM to achieve both of the above goals.

Literature Review

Much of the research on order batching in warehouse picking has focused on minimizing travel distance or travel time of order pickers or AS/RS machines. Ho & Tseng (2006) minimize total travel distance for order pickers in two cross-aisles and Chen, Chenc, & Hsu (2005) minimize total travel distance in any layout. Huang, Liou, & Tsai (2008) extend the research by minimizing total travel distance while considering order due time also. Gibson & Sharp (1992) reduce tour lengths in an order retrieval system and Baek, Hwang, & Lee (1988) minimize total distance traveled by the machine in an AS/RS. De Koster, Poort, & Wolters (1999) minimize total travel time of order pickers and Gademann & Van De Velde (2001) minimize total travel time of order pickers in parallel aisles. Hwang & Lee (1988) minimize total travel time of a man-on-board AS/RS. Olafson and Won (2005) improve efficiency measured by low picking time while optimizing customer response time.

A smaller amount of research has focused on other metrics in order picking efficiency. Gademann, Van Den Berg, & Van Der Hoff (2005) determine order batches to minimize maximum lead time for wave picking in a parallel aisle warehouse. Elsayed & Lee (1996) minimized total tardiness of retrievals with AR/RS systems and Elsayed, Kim, Lee, & Scherer (1993) introduced a penalty function for earliness and tardiness.

In the past research on warehouse systems without AS/RS systems, the order batches were constructed based upon a metric related to the travel of order pickers throughout the warehouse. The research on warehouse systems with AS/RS systems either focuses on metrics related to the movement of the AS/RS or the tardiness of orders. The research in this paper is similar to past research yet varies from anything that has been done. This research seeks to minimize the pick labor time while satisfying the on-time deliveries with an AS/RS system. No past research has selected a batching schedule that guarantees on-time delivery while minimizing pick costs!

Modeling Approach

Since the pick process at the IBM Replenishment Plant is nonlinear in nature, the process is first modeled using a nonlinear formulation. A nonlinear math programming formulation is used as a modeling approach to minimize pick and holding costs in the process while satisfying all demands. The formulation uses a nonlinear pick rate function to determine the time to pick a defined number of parts for a product. This nonlinear pick rate function is used to determine pick costs in the objective function. Of course, the order sizes and order completion times are used to satisfy demand and calculate holding costs.

The nonlinear math programming formulation with the nonlinear pick rate function ensures that all pick demand times for all products are satisfied while minimizing pick and holding costs. The assumptions made in modeling the operation are:

- Single AS/RS
- Multiple products
- Pick schedule for one day
- Deterministic demand
- Deterministic pick times
- Demand occurs only at pre-determined intervals
- Demand schedule is feasible
- No late orders allowed

The model is:

Model 1

Sets:

i: time periods
j: alias set to the set i
p: set of products

Data:

C: pick cost for one time period
K: cost to start a pick
D_{ip}: demand at the beginning of period i for product p
h_p: holding cost of one unit for one time period for product p
$F(X_{ip})$: amount of labor time to pick an amount of size X_i for product p
n: number of periods in the model
P: number of products

Decision Variables:

X_{ip}: pick size amount finished at time i for product p
Y_{jp}: equals 1 when a pick is finished at time j and 0 otherwise for product p
S_{jp}: surplus of pick amount over demand at time j for product p

Formulation:

$$\min_{X_{ip},Y_{jp},S_{jp}} \sum_{i=1}^{n}\sum_{p=1}^{P}(C*F(X_{ip}))+\sum_{j=1}^{n}\sum_{p=1}^{P}(K*Y_{jp})+\sum_{p=1}^{P}h_p\sum_{j=1}^{n}S_{jp} \quad (1)$$

$$\sum_{i=1}^{j}(X_{ip}-D_{ip})=S_{jp} \qquad \forall j=1,\ldots,n;\ p=1,\ldots,P \quad (2)$$

$$i-F(X_{ip})\geq 0 \qquad \forall i=1,\ldots,n;\ p=1,\ldots,P \quad (3)$$

$$X_{ip}-Y_{jp}M\leq 0 \qquad \forall i=1,\ldots,n;\ j=1,\ldots,n; i=j;\ p=1,\ldots,P \quad (4)$$

$$(j-(i-F(X_{ip})))+nY_{jp}\leq n \qquad \forall i=1,\ldots,n;\ j<i;\ p=1,\ldots,P \quad (5)$$

$$Y_{jp}=0,1 \qquad \forall j=1,\ldots,n;\ p=1,\ldots,P \quad (6)$$

$$S_{jp}\geq 0 \qquad \forall j=1,\ldots,n;\ p=1,\ldots,P \quad (7)$$

$$X_{ip}\geq 0 \qquad \forall i=1,\ldots,n;\ p=1,\ldots,P \quad (8)$$

Equation 1 minimizes the pick, pick startup, and holding costs for all products.

Constraint 2 insures that the amount of parts picked by time j satisfies total demand by time j for each product p.

Constraint 3 requires that the labor time to pick an amount finishing at time i for product p is less than the time from time 0 to time i.

Constraint 4 sets Y =0 if an order is not finished at time i and to 1 if an order is finished at time i.

Constraint 5 insures that no more than one order is open at any point in time.

On inspection this formulation has a nonlinear objective function, nonlinear constraints, and binary decision variables which make the model difficult to solve. Sahinidis and Tawarmalani (2004) show the difficulty in solving a nonlinear integer program. The model would be easier to solve without the nonlinear objective function and constraints. If a linear math programming model could be formulated that approximates the nonlinear model, the solution method might be more efficient.

Harrison and Neidigh (2009) successfully solved a nonlinear production scheduling problem using a piecewise linear approximation. The nonlinear objective function and constraints in the production scheduling problem are replaced with linear equations through the use of discrete scheduling periods. Production can only occur during these discrete periods or over multiple consecutive discrete periods. Binary variables are used to control production during the discrete periods. As the discrete time periods become small the solution to the linear approximation problem comes close to the solution of the nonlinear problem. Fortunately the properties of the production scheduling problem solved by Harrison and Neidigh are very similar to those of the nonlinear pick scheduling problem modeled here.

Thus, the nonlinear pick process can be reformulated using the piecewise linear approximation model of Harrison and Neidigh. In this formulation, picks occur during discrete time periods such as time i to time j. During these time periods the pick amount is fixed per the nonlinear function $F(X_{ip})$. The piecewise linear approximation formulation can be adapted to any nonlinear pick rate function as long as the pick amount can be determined for any discrete time period. The mini-pick operation was modeled with the linear approximation model based upon the following assumptions:

- Single AS/RS
- Multiple products
- Pick schedule for one day
- Deterministic demand
- Deterministic pick times
- Demand occurs only at pre-determined intervals
- Demand schedule is feasible
- No late orders allowed

Model 2

Sets:

i: beginning time period in a pick interval

j: ending time period in a pick interval

k: alias set of j

p: set of products

Data:

C: pick cost for one time period

K: cost to start a pick

D_{jp}: demand at time period j for product p

h_p: holding cost for one unit for one time period for product p

n: number of periods in the model

P: number of products

P_{ijp}: amount picked from time period i to j for product p

Decision Variables:

X_{ijp}: 0 if not picking from time period *i* to *j* for product *p*

1 if picking from time period *i* to *j* for product *p*

S_{kp}: surplus of pick amount over demand at time *k* for product *p*

Mini-Pick Multi-Product Single-Machine Formulation:

$$Min \sum_{p=1}^{P}\sum_{j=1}^{n}\sum_{i=0}^{j-1} C(j-i)X_{ijp} + \sum_{p=1}^{P}\sum_{j=1}^{n}\sum_{i=0}^{j-1} K * X_{ijp} + \sum_{p=1}^{P} h_p \sum_{k=1}^{n} S_{kp} \qquad (1)$$

$$\sum_{j=1}^{k}\sum_{i=0}^{j-1} P_{ijp} X_{ijp} - S_{kp} = \sum_{j=1}^{k} D_{jp} \qquad \forall k = 1,\ldots,n;\ p = 1,\ldots,P \qquad (2)$$

$$\sum_{p=1}^{P}\sum_{i=0}^{k-1}\sum_{j=k}^{n} X_{ijp} \leq 1 \qquad \forall k = 1,\ldots,n \qquad (3)$$

$$S_{kp} \geq 0 \qquad \forall k = 1,\ldots,n;\ p = 1,\ldots,P \qquad (4)$$

$$X_{ijp} = 0,1 \qquad \forall i < j;\ j = 1,\ldots,n;\ p = 1,\ldots,P \qquad (5)$$

Equation 1 minimizes the pick, pick startup, and holding costs for all products.

Constraint 2 insures that each demand for each product is satisfied and calculates the excess demand at each time period.

Constraint 3 insures that no more than one pick is occurring during a time period.

Constraint 4 sets all surplus variables to nonnegative numbers.

Constraint 5 defines the decision variables that determine picks during certain intervals. If a variable is equal to 1, then a pick will occur and if a variable is equal to 0 then no pick will occur.

Model Testing

In order for the model to be considered a success, the model must demonstrate the ability to construct pick schedules that handle the following scenarios:

- Schedule picks for multiple product demands that minimize holding costs
- Schedule multiple picks that in total satisfy one demand
- Aggregate multiple demands into one pick

We constructed several two-product ten-period pick schedules to test the above scenarios. The model was formulated and tested using GAMS [7]. The following parameters were used for both products for all testing:

Order start cost: $15.00

Order pick cost for one time period: $75.00

$P_{ijp} = [(j - i)/0.4]^{(1/0.65)}$

The holding costs for both products were varied in order to test the above scheduling scenarios. The following demand schedules were tested:

Schedule 1:

Product 1 – Demand of 40 in Period 9 and Holding cost of 0.5/period

Product 2 – Demand of 20 in Period 8 and Holding cost of 5/period

Optimum solution – Pick Product 1 from Periods 0 to 5 and Product 2 from Periods 5 to 8

Schedule 2:

Product 1 – Demand of 40 in Period 9 and Holding cost of 0.5/period

Product 2 – Demand of 20 in Period 8 and Holding cost of 0.5/period

Optimum solution – Pick Product 1 from Periods 4 to 9 and Product 2 from Periods 1 to 4

Schedules 1 & 2 are similar except the holding costs vary for Product 2. When the holding cost is large for Product 2 in relation to the Product 1 holding cost, Product 2 is picked after Product 1 to minimize the holding costs. When the two holding costs are equal, Product 1 is picked later because the demand for Product 1 occurs later than the demand for Product 2. These results demonstrate the ability of the model to vary pick lots to minimize holding costs.

Schedule 3:

Product 1 – Demand of 39 in Period 9 and Holding cost of 5/period

Product 2 – Demand of 20 in Period 8 and Holding cost of 15/period

Optimum solution – Pick Product 1 from Periods 1 to 5 and 8 to 9, Product 2 from Periods 5 to 8

The solution to Schedule 3 has split the picking of the demand at Period 9 for Product 1 into two pick lots. This occurs due to the large holding costs for each product and demonstrates the ability of the model to split pick lots as needed.

Schedule 4:

Product 1 – Demand of 20 in Period 7 and 20 in Period 9 and Holding cost of 0.5/period

Product 2 – Demand of 20 in Period 8 and Holding cost of 0.5/period

Optimum solution – Pick Product 1 from Periods 0 to 5 and Product 2 from Periods 5 to 8

Schedule 5:

Product 1 – Demand of 20 in Period 7 and 20 in Period 9 and Holding cost of 5/period

Product 2 – Demand of 20 in Period 8 and Holding cost of 0.5/period

Optimum solution – Pick Product 1 from Periods 3 to 6 and 6 to 9, Product 2 from Periods 0 to 3

In Schedule 4 the two demands for Product 1were aggregated into one pick and in Schedule 5 the two demands for Product 1 were picked separately. These schedules demonstrate the ability of the model to aggregate multiple demands as necessary to minimize costs.

These tests show that this model can successfully schedule each of the above scenarios so this model can be used to schedule the nonlinear pick rate!

Conclusions

The linear approximation model for the mini-pick operation successfully removes the nonlinear objective function and nonlinear constraints in Model 1. This is achieved because the nonlinear pick rate function is removed from the model. Instead, the number of orders that can be picked for any discrete time period for any product is determined using the nonlinear pick rate function and entered as data into the problem. As long as the nonlinear pick rate function can be used to determine the number of orders picked in any discrete time period, this linear approximation formulation can be used to replace and approximate the nonlinear model.

Our tests show that the linear approximation model can appropriately schedule pick lots for the following three scenarios: schedule picks for multiple product demands that minimize holding costs, schedule multiple picks that in total satisfy one demand, and aggregate multiple demands into one pick. Thus, this linear approximation formulation can be successfully used to model the mini-pick scheduling problem!

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Biographies

Robert O. Neidigh is an assistant professor in the Department of Supply Chain Management of the John L. Grove College of Business at Shippensburg University. He holds a PhD in Supply Chain Management from the Pennsylvania State University. His research interests include production scheduling, math programming, and order picking. He has published in several academic journals including *International Journal of Production Research* and the *Journal of Business Cases and Applications* among others.

Allison Shirley is a 2008 graduate of Shippensburg University with a BS in Business Administration major in Supply Chain Management and minor in Economics. She was a member of the Varsity field hockey program for 4 years. Allison worked for IBM in 2008 as a SAP MM Consultant, and she is currently working as a Procurement Specialist for The Boeing Company.